# L2 – Integration to notes

# Contents: Lathi chap. 2

- Time-domain analysis of Continuous time systems

  System response = Zero-state response + Zero-input response

  - Zero-state response
    - The initial conditions are all zero: $y(t)=f(t)*h(t)$
    - Convolution integral
  - Zero-input response
    - The input signal is zero: $y(t)=y_0(t)$=linear combination of system modes
  - Unit impulse response
    - The input signal is the delta function: $h(t)$

# Zero-state response

$$y(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)\,d\tau$$

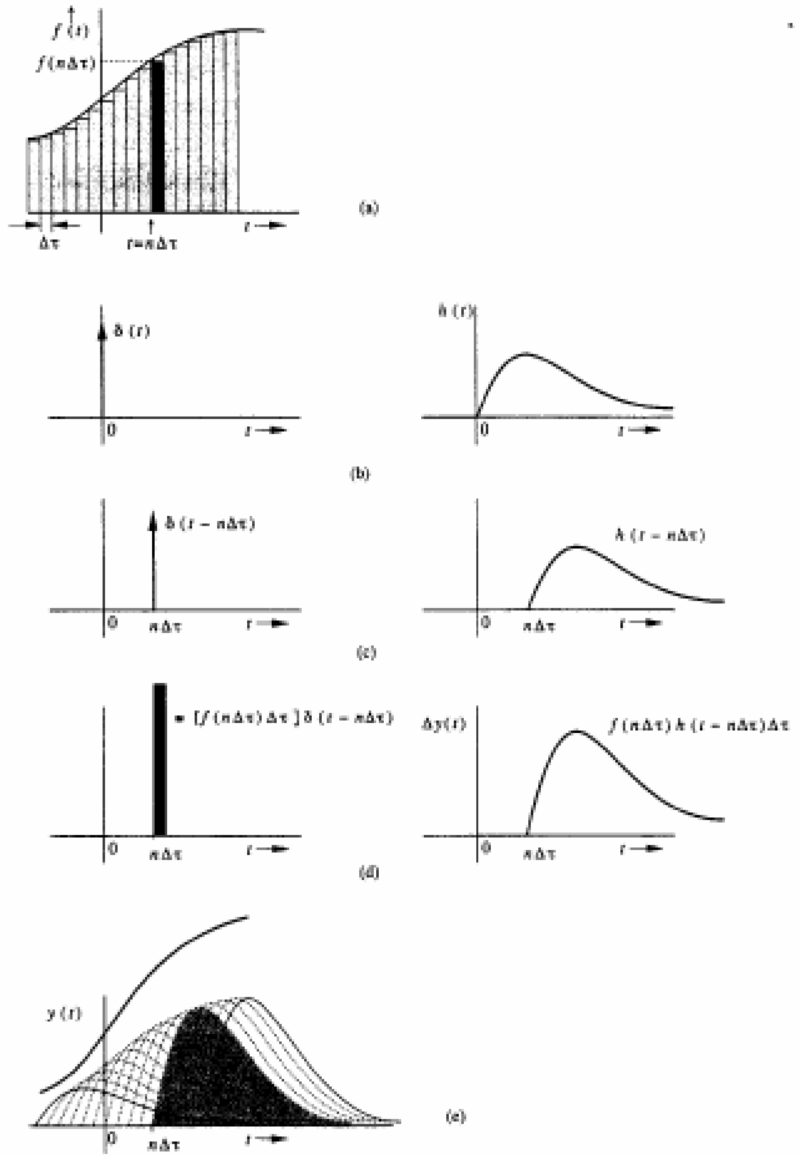



Fig. 2.3 Finding the system response to an arbitrary input $f(t)$.

# Zero-input response

- Real roots

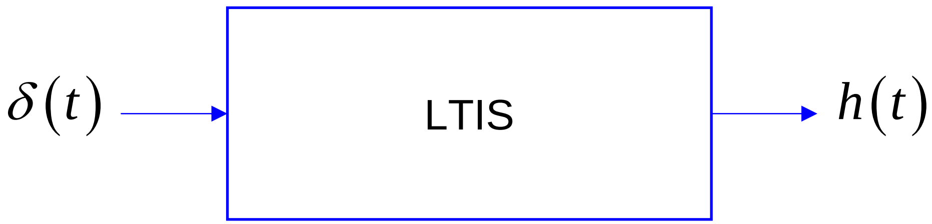
$$y_0(t) = (c_1 + c_2 t + \cdots + c_r t^{r-1}) e^{\lambda_1 t} + c_{r+1} e^{\lambda_{r+1} t} + \cdots + c_n e^{\lambda_n t}$$

repeated with multiplicity r

- Complex conjugate roots

$$\begin{aligned} y_0(t) &= \frac{c}{2} e^{j\theta} e^{(\alpha + j\beta)t} + \frac{c}{2} e^{-j\theta} e^{(\alpha - j\beta)t} \\ &= \frac{c}{2} e^{\alpha t} \left[ e^{j(\beta t + \theta)} + e^{-j(\beta t + \theta)} \right] \\ &= c e^{\alpha t} \cos(\beta t + \theta) \end{aligned}$$

# Unit impulse response




$$h(t) = \text{characteristic mode terms} \qquad t \geq 0^{+}$$

$$h(t) = b_n \delta(t) + [P(D) y_n(t)] u(t)$$

$y_n(t)$ is a “special case” of $y_0(t)$ when a particular set of initial conditions hold.
These conditions account for the fact that the system state was perturbed by a delta in t=0

# Unit impulse response

where $b_n$ is the coefficient of the $n$th-order term in $P(D)$ [see Eq. (2.17b)], and $y_n(t)$ is a linear combination of the characteristic modes of the system subject to the following initial conditions:

$$y_n^{(n-1)}(0) = 1, \quad \text{and} \quad y_n(0) = \dot{y}_n(0) = \ddot{y}_n(0) = \cdots = y_n^{(n-2)}(0) = 0 \qquad (2.20)$$

where $y_n^{(k)}(0)$ is the value of the $k$th derivative of $y_n(t)$ at $t = 0$. We can express this condition for various values of $n$ (the system order) as follows:

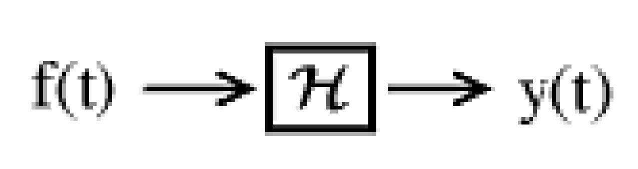
$$\begin{aligned} n = 1: &\ y_n(0) = 1 \\ n = 2: &\ y_n(0) = 0 \text{ and } \dot{y}_n(0) = 1 \\ n = 3: &\ y_n(0) = \dot{y}_n(0) = 0 \text{ and } \ddot{y}_n(0) = 1 \\ n = 4: &\ y_n(0) = \dot{y}_n(0) = \ddot{y}_n(0) = 0 \text{ and } \dddot{y}_n(0) = 1 \end{aligned} \qquad (2.21)$$

and so on.

If the order of $P(D)$ is less than the order of $Q(D)$, $b_n = 0$, and the impulse term $b_n\delta(t)$ in $h(t)$ is zero.

# Continuous time linear systems

- Physically realizable, linear time-invariant systems can be described by a set of linear dierential equations (LDEs):

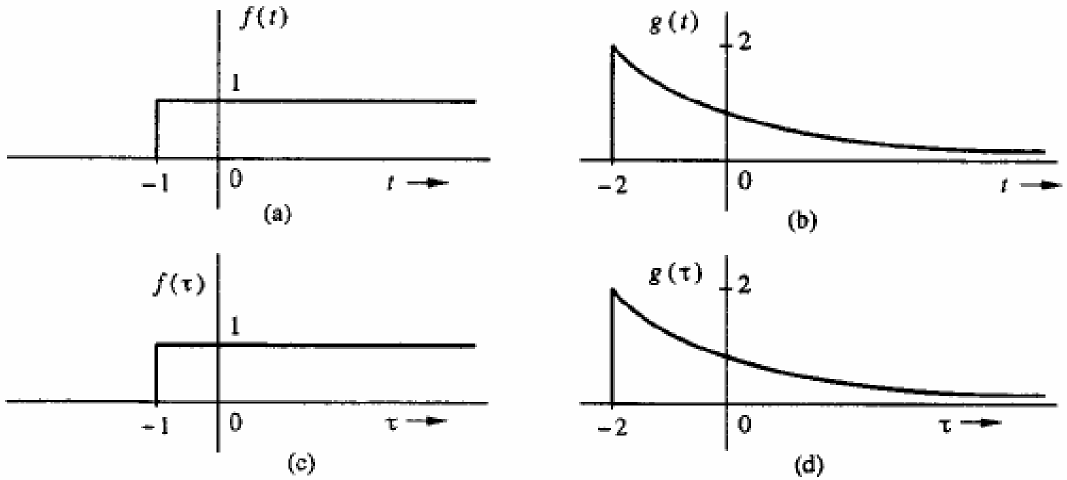


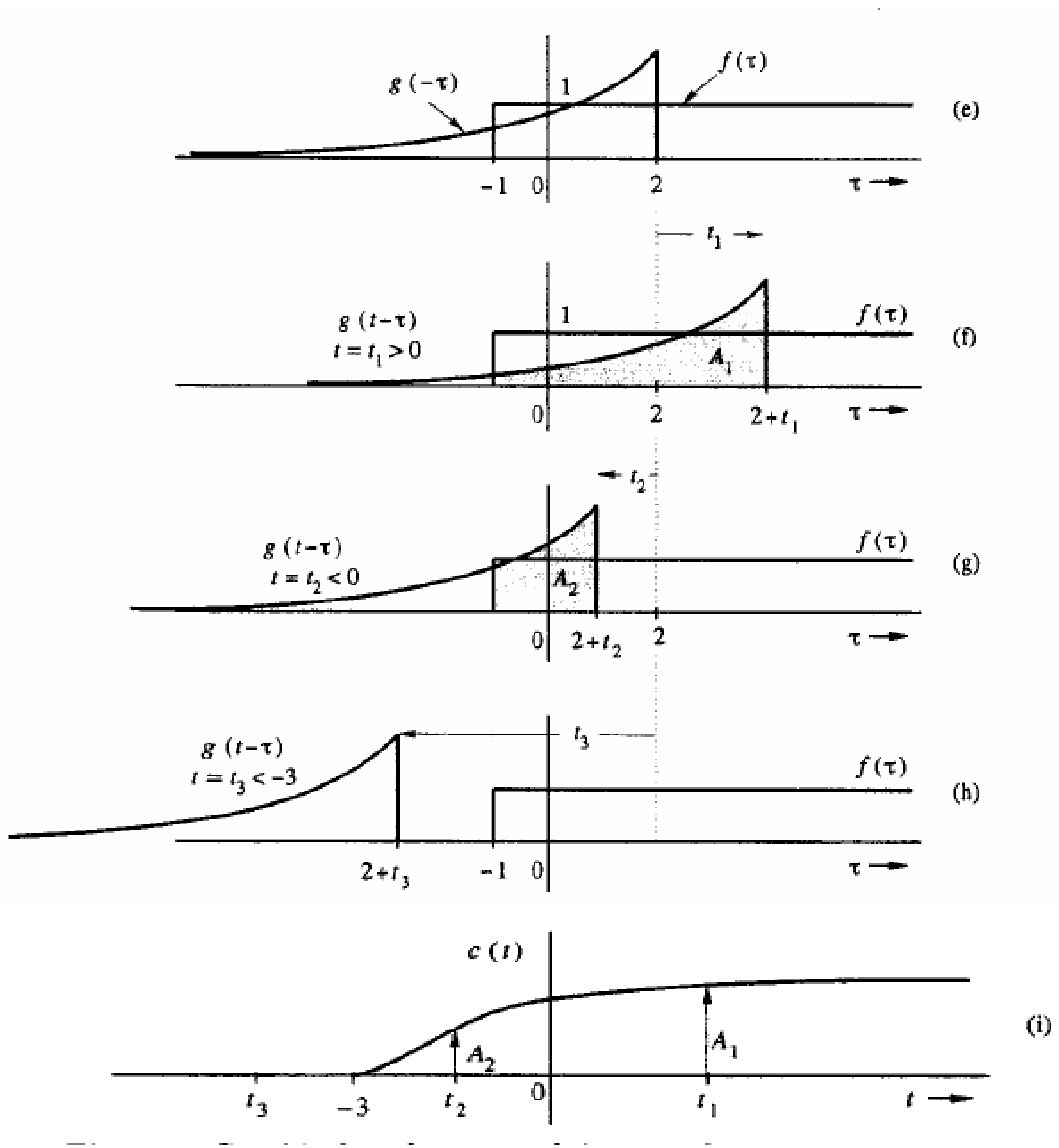

$$\frac{d^n}{dt^n}y(t) + a_{n-1}\frac{d^{n-1}}{dt^{n-1}}y(t) + \cdots + a_1\frac{d}{dt}y(t) + a_0 y(t) = b_m\frac{d^m}{dt^m}f(t) + \cdots + b_1\frac{d}{dt}f(t) + b_0 f(t)$$

$$\sum_{i=0}^{n}\left(a_i\frac{d^i}{dt^i}y(t)\right) = \sum_{i=0}^{m}\left(b_i\frac{d^i}{dt^i}f(t)\right)$$

# Continuous time convolution

$$f(t)*g(t)=\int_{-\infty}^{+\infty} f(\tau)g(t-\tau)d\tau$$




g (−τ)
1
f (τ)
(e)
−1
0
2
τ
t₁
g (t−τ)
t = t₁ > 0
1
f (τ)
(f)
A₁
0
2
2+t₁
τ
t₂
g (t−τ)
t = t₂ < 0
f (τ)
(g)
A₂
0
2+t₂
2
τ
g (t−τ)
t = t₃ < −3
t₃
f (τ)
(h)
2+t₃
−1
0
τ
c (t)
(i)
A₂
A₁
t₃
−3
t₂
0
t₁
t

# Summary

## Summary of the Graphical Procedure

The procedure for graphical convolution can be summarized as follows:

1. Keep the function $f(\tau)$ fixed.
2. Visualize the function $g(\tau)$ as a rigid wire frame, and rotate (or invert) this frame about the vertical axis ($\tau = 0$) to obtain $g(-\tau)$.
3. Shift the inverted frame along the $\tau$ axis by $t_0$ seconds. The shifted frame now represents $g(t_0 - \tau)$.
4. The area under the product of $f(\tau)$ and $g(t_0 - \tau)$ (the shifted frame) is $c(t_0)$, the value of the convolution at $t = t_0$.
5. Repeat this procedure, shifting the frame by different values (positive and negative) to obtain $c(t)$ for all values of $t$.