

14. Let Σ be the axiom system for the fields of characteristic 0, as defined for 13. Suppose that there were an axiom system Γ for the fields of finite characteristic. Since $\Gamma \cup \Sigma$ is not satisfiable (no field can be both of finite char. and of char. 0), by the Compactness theorem there are finite $\Sigma_0 \subseteq \Sigma, \Gamma_0 \subseteq \Gamma$ such that $\Gamma_0 \cup \Sigma_0$ is not satisfiable; whence $\Gamma \cup \Sigma_0$ is not satisfiable either.

If $\Sigma_0 = \{A_1, \dots, A_n\}$ and A is $A_1 \wedge \dots \wedge A_n$, then A is a sentence with $\Sigma \models A$ such that $\Gamma \cup \{A\}$ is not satisfiable, i.e. $\Gamma \models \neg A$ (check by unfolding the def. of $\Gamma \models \neg A$).

In all, A holds in all fields of char. 0, but fails in every field of finite char. (in which actually $\neg A$ holds); this is impossible in view of 13.