

Applicando il cambiamento di variabile:

$$\iint_T xy \, dx \, dy = \iint_K \rho \cos \theta \rho \sin \theta \cdot \underbrace{\rho}_{|J(\phi)|} \, d\rho \, d\theta = \iint_K \rho^3 \cos \theta \sin \theta \, d\rho \, d\theta = \int \rho^3 \sin 2\theta \, d\rho \, d\theta$$

$$= \frac{1}{2} \iint_K \rho^3 \sin(2\theta) \, d\rho \, d\theta = \frac{1}{4} \iint_K \rho^3 2 \sin(2\theta) \, d\rho \, d\theta =$$

$$= + \frac{1}{4} \int_0^1 d\rho \int_{\frac{7\pi}{4}}^{2\pi} \rho^3 2 \sin(2\theta) \, d\theta = - \frac{1}{4} \left[\int_0^1 \rho^3 \left[\cos(2\theta) \right]_{\frac{7\pi}{4}}^{2\pi} d\rho \right] =$$

$$= - \frac{1}{4} \int_0^1 \rho^3 \left(\underbrace{\cos(4\pi)}_1 - \underbrace{\cos\left(\frac{7}{2}\pi\right)}_0 \right) d\rho = - \frac{1}{4} \int_0^1 \rho^3 \, d\rho = - \frac{1}{4} \left[\frac{\rho^4}{4} \right]_0^1 =$$

$$= - \frac{1}{16}$$