Multiresolution analysis

Exercise Session 5

Exercise 1: Wavelet transform

Build an image that corresponds to a polynomial 2D function (N=M=512) with maximum degree equal to 4. For the resulting image and for the image `mandrill.tif`

1. Perform the Discrete Wavelet transform with the parameters listed in the Table and visually compare the results.

Number of levels	J=4
Filters	haar, biorthogonal 2.2, biorthogonal 2.4, Daubechies' 4 (db4)

2. Choose one wavelet killing the maximum degree polynomial appearing in your image. Set $J = 3$. Perform the following operations on the subband coefficients:

   (a) Successively set to zero the different subband coefficients (first subband by subband and then level by level);

   (b) Keep the absolute value of the coefficients (change each negative sign to positive);

   (c) Keep the signs of the coefficients while setting the absolute value of each coefficient equal to $(\sqrt{2})^j$, where j represents the level;

   (d) Quantize the coefficients by rounding to the nearest integer;

   (e) Quantize the coefficients of the different subbands such that the quantization step q_j is given by

   $$q_j = \lfloor \frac{\max_{n,m} |c_{j,n,m}|}{N} \rfloor \tag{1}$$

   for $N = 1, \ldots, 4$;

   (f) For each N, perform the inverse transform to reconstruct the image and calculate the PSNR between the original and the reconstructed image;

   (g) Plot the PSNR as a function of N;

$$PSNR = 20 \log_{10} \frac{255}{\sqrt{MSE}} \tag{2}$$

$$MSE = \frac{\sum_i \sum_j (im1[i,j] - im2[i,j])^2}{N_x N_y} \tag{3}$$

Comment the results.

Exercise 2: Subband statistics

For each of the following images: `einstein.jpg, mandrill.tif, Flowers.003.tif`

1. Plot the histogram of the image (using 128 bins);
2. Perform the DWT and the SWT with $J = 3$ and filter 'sym2';
3. Plot the histogram of each subband using the same number of bins;
4. Comment the results. Hint: note that the `imhist` function requires a normalized input (values between zero and 1).