Applicando il cambiamento di variabile

$$\iint_T xy\,dxdy = \iint_K \rho\cos\vartheta\,\rho\sin\vartheta\,\underbrace{\rho}_{|J(\Phi)|}\,d\rho d\vartheta = \iint_K \rho^3\cos\vartheta\sin\vartheta\,d\rho d\vartheta =$$

$\sin 2\vartheta = 2\sin\vartheta\cos\vartheta$

$$= \frac{1}{2}\iint_K \rho^3 \sin(2\vartheta)\,d\rho\,d\vartheta = \frac{1}{4}\iint_K \rho^3\, 2\sin(2\vartheta)\,d\rho d\vartheta =$$

$$= +\frac{1}{4}\int_0^1 d\rho \int_{\frac{3}{4}\pi}^{\pi} \rho^3\, 2\sin(2\vartheta)\,d\vartheta = -\frac{1}{4}\left(\int_0^1 \rho^3 \left(\cos(2\vartheta)\Big|_{\frac{3}{4}\pi}^{\pi} d\rho\right)\right) =$$

$$= -\frac{1}{4}\int_0^1 \rho^3 \Big(\underbrace{\cos 2\pi}_{=1} - \underbrace{\cos \tfrac{3}{2}\pi}_{=0}\Big)\,d\rho = -\frac{1}{4}\int_0^1 \rho^3 d\rho = -\frac{1}{4}\,\frac{\rho^4}{4}\Big|_0^1 =$$

$$= -\frac{1}{16}$$

EX3 foglio D parte 2