Considero

$$0 \le \frac{(xy)^2}{x^2+y^2} \le \frac{1}{2}\,\frac{(x^4+y^4)}{x^2+y^2} \le \frac{1}{2}\,\frac{x^4+2x^2y^2+y^4}{x^2+y^2} = \frac{1}{2}\,\frac{(x^2+y^2)^2}{x^2+y^2} = \frac{x^2+y^2}{2}$$

$$0 \le (x^2-y^2)^2 = x^4+y^4-2x^2y^2$$

$$x^2y^2 \le \frac{1}{2}(x^4+y^4)$$

$$x^2y^2 \ge 0$$

$$0 \le \frac{(xy)^2}{x^2+y^2} \xrightarrow{(x,y)\to(0,0)} 0 \qquad \frac{x^2+y^2}{2} \xrightarrow{(x,y)\to(0,0)} 0$$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{(xy)^2}{x^2+y^2} = 0$$

Quindi

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{\cos(xy)-1}{(xy)^2} \cdot \frac{(xy)^2}{x^2+y^2} = 0$$

$$\frac{\cos(xy)-1}{(xy)^2} \to -\frac{1}{2} \qquad \frac{(xy)^2}{x^2+y^2} \to 0$$

per il teorema del limite del prodotto