Data-intensive computing systems

Graph Algorithms

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Acknowledgements

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  - *Part of the course material is based on slides provided by the following authors*
    - *Pietro Michiardi, Jimmy Lin*

What's a graph?

- G = (V,E), where
  - V represents the set of vertices (nodes)
  - E represents the set of edges (links)
  - Both vertices and edges may contain additional information
- Different types of graphs:
  - Directed vs. undirected edges
  - Presence or absence of cycles
- Graphs are everywhere:
  - Hyperlink structure of the web
  - Physical structure of computers on the Internet
  - Interstate highway system
  - Social networks

Some Graph Problems

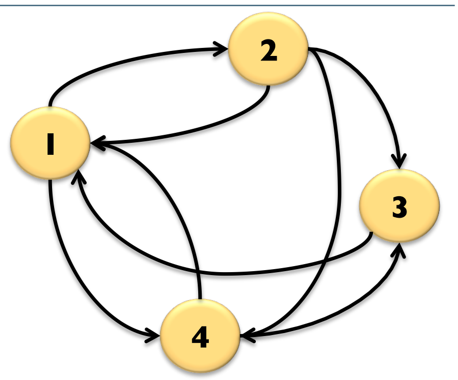
- Finding shortest paths
  - Routing Internet traffic and UPS trucks
- Finding minimum spanning trees
  - Telco laying down fiber
- Finding Max Flow
  - Airline scheduling
- Identify "special" nodes and communities
  - Breaking up terrorist cells, spread of avian flu
- Bipartite matching
  - Monster.com, Match.com
- And of course... PageRank

Graphs and MapReduce

- A large class of graph algorithms involve:
  - Performing computations at each node: based on node features, edge features, and local link structure
  - Propagating computations: “traversing” the graph
- Key questions:
  - How do you represent graph data?
    - Adjacency matrix
    - Adjacency list
  - How do you traverse a graph in MapReduce?

Adjacency Matrices

- Represent a graph as an n x n square matrix M
  - $n = |V|$
  - $M_{ij} = 1$ means a link from node i to j



	1	2	3	4
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0

- Advantages:
  - Amenable to mathematical manipulation
  - Iteration over rows and columns corresponds to computations on outlinks and inlinks
- Disadvantages:
  - Lots of zeros for sparse matrices
  - Lots of wasted space

Adjacency Lists

- ❑ Take adjacency matrices… and throw away all the zeros

	1	2	3	4
1	0	1	0	1
2	1	0	1	1
3	1	0	0	0
4	1	0	1	0

1: 2, 4
2: 1, 3, 4
3: 1
4: 1, 3

- ❑ Advantages:
  - Much more compact representation
  - Easy to compute over outlinks
- ❑ Disadvantages:
  - Much more difficult to compute over inlinks

Agenda

- ❑ Graph algorithms in MapReduce
  - Parallel breath-first search
  - PageRank
- ❑ Graph Analysis Beyond Mapreduce
  - Pregel

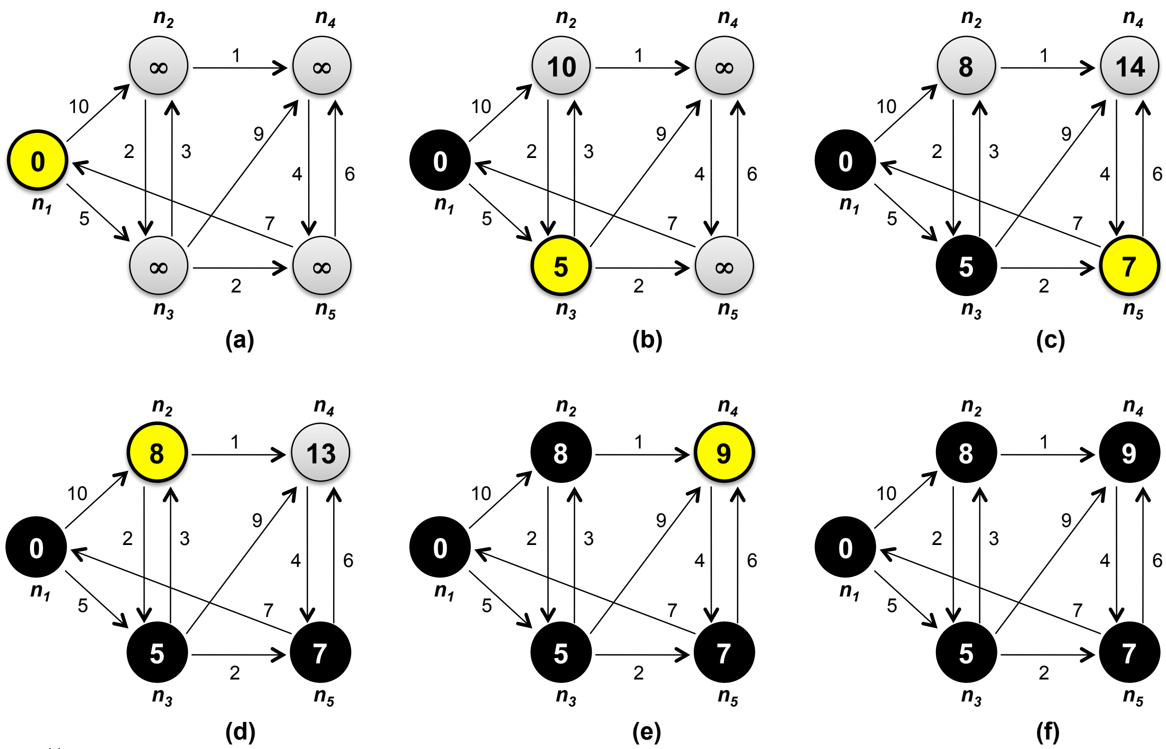
Parallel Breath-First Search

Single-Source Shortest Path (SSSP)

- Problem
  - Find shortest path from a source node to all target nodes
- Solution on a single machine
  - Dijkstra algorithm using a global priority queue
    - Maintains a globally sorted list of nodes by current distance

$\textsc{Dijkstra}(G, w, s)$
  $d[s] \leftarrow 0$
  for all vertex $v \in V$ **do**
    $d[v] \leftarrow \infty$
  $Q \leftarrow \{V\}$
  while $Q \neq \emptyset$ **do**
    $u \leftarrow \textsc{ExtractMin}(Q)$
    for all vertex $v \in u.\textsc{AdjacencyList}$ **do**
      if $d[v] > d[u] + w(u, v)$ **then**
        $d[v] \leftarrow d[u] + w(u, v)$

Example: SSSP – Dijkstra's Algorithm



SSSP on large instances

- How to solve this problem in parallel?
  - "Brute-force" approach: breadth-first search (BFS)

- Parallel BFS: intuition
  - Flooding
  - Iterative algorithm in MapReduce
  - Try to mimic message passing style algorithms

Parallel Breadth-First Search: Pseudo code

class MAPPER
    method MAP(nid n, node N)
        $d \leftarrow N$.DISTANCE
        EMIT(nid n, N) ▷ Pass along graph structure
        for all nodeid $m \in N$.ADJACENCYLIST **do**
            EMIT(nid $m, d+1$) ▷ Emit distances to reachable nodes

class REDUCER
    method REDUCE(nid $m, [d_1, d_2, \ldots]$)
        $d_{min} \leftarrow \infty$
        $M \leftarrow \emptyset$
        for all $d \in$ counts $[d_1, d_2, \ldots]$ **do**
            if ISNODE(d) **then**
                $M \leftarrow d$ ▷ Recover graph structure
            else if $d < d_{min}$ **then** ▷ Look for shorter distance
                $d_{min} \leftarrow d$
        M.DISTANCE $\leftarrow d_{min}$ ▷ Update shortest distance
        EMIT(nid m, node M)

Parallel Breadth-First Search

- Assumptions
  - Connected, directed graph
  - Data structure: adjacency list
  - Distance to each node is stored alongside the adjacency list of that node
- The pseudo-code
  - We use n to denote the node id (an integer)
  - We use N to denote the node adjacency list and current distance
  - The algorithm works by mapping over all nodes
  - Mappers emit a key-value pair for each neighbor on the node’s adjacency list
    - The key: node id of the neighbor
    - The value: the current distance to the node plus one
    - If we can reach node n with a distance d, then we must be able to reach all the nodes connected to n with distance d + 1

Parallel Breadth-First Search

- The pseudo-code (continued)
  - After shuffle and sort, reducers receive keys corresponding to the destination node ids and distances corresponding to all paths leading to that node
  - The reducer selects the shortest of these distances and update the distance in the node data structure

- Passing the graph along
  - The mapper: emits the node adjacency list, with the node id as the key
  - The reducer: must distinguish between the node data structure and the distance values

Parallel Breadth-First Search

- MapReduce iterations
  - The first time we run the algorithm, we “discover” all nodes connected to the source
  - The second iteration, we discover all nodes connected to those
  - → Each iteration expands the “search frontier” by one hop
  - How many iterations before convergence?

- This approach is suitable for small-world graphs
  - The diameter of the network is small

Parallel Breadth-First Search

- Checking the termination of the algorithm
  - Requires a "driver" program which submits a job, check termination condition and eventually iterates
  - In practice:
    - Hadoop counters
    - Side-data to be passed to the job configuration
- Extensions
  - Storing the actual shortest-path
  - Weighted edges (as opposed to unit distance)

Summary

- The graph structure is stored in an adjacency lists
  - This data structure can be augmented with additional information
- The MapReduce framework
  - Maps over the node data structures involving only the node's internal state and it's local graph structure
  - Map results are "passed" along outgoing edges
  - The graph itself is passed from the mapper to the reducer
    - This is a very costly operation for large graphs!
  - Reducers aggregate over "same destination" nodes
- Graph algorithms are generally iterative
  - Require a driver program to check for termination

PageRank

Graph algorithm: PageRank

- What is PageRank
  - It's a measure of the relevance of a Web page, based on the structure of the hyperlink graph
  - Based on the concept of random Web surfer
- Formally we have:

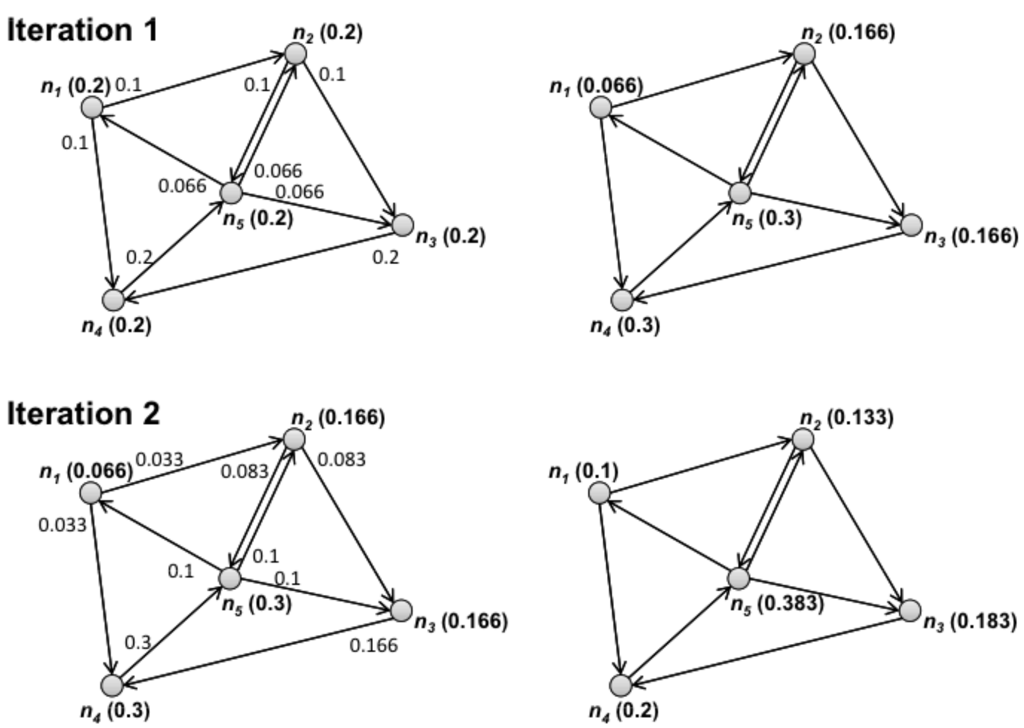
$$P(n) = \alpha\left(\frac{1}{|G|}\right) + (1-\alpha)\sum_{m \in L(n)} \frac{P(m)}{C(m)}$$

  - |G| is the number of nodes in the graph
  - α is a random jump factor
  - L(n) is the set of out-going links from page n
  - C(m) is the out-degree of node m

PageRank in Details

- PageRank is defined recursively, hence we need an iterative algorithm
  - A node receives “contributions” from all pages that link to it
- Consider the set of nodes L(n)
  - A random surfer at m arrives at n with probability 1/C(m)
  - Since the PageRank value of m is the probability that the random surfer is at m, the probability of arriving at n from m is P(m)/C(m)
- To compute the PageRank of n we need:
  - Sum the contributions from all pages that link to n
  - Take into account the random jump, which is uniform over all nodes in the graph

PageRank: Example

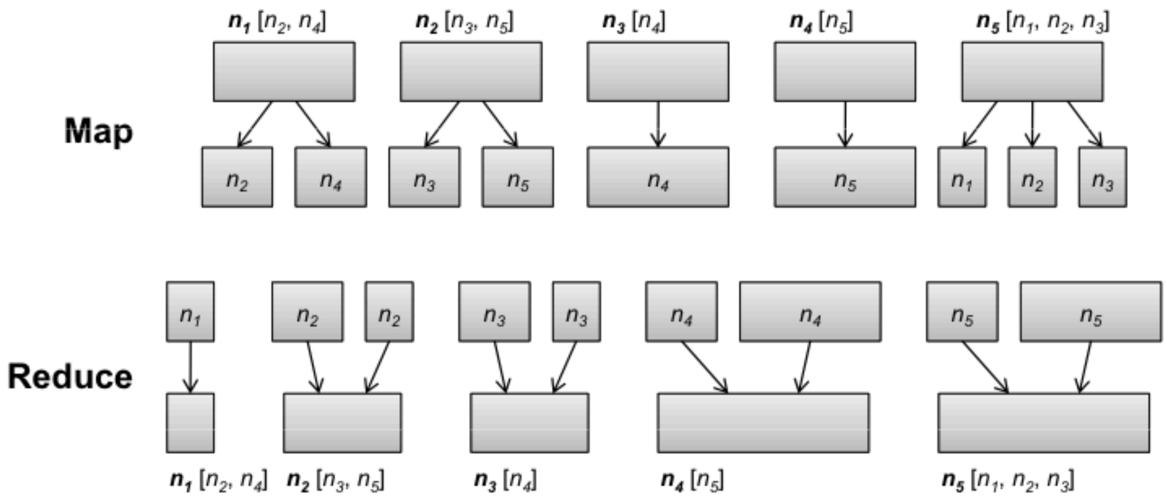


PageRank: pseudo-code

```
class Mapper
    method Map(nid n, node N)
        p ← N.PageRank/|N.AdjacencyList|
        Emit(nid n, N)                              ▷ Pass along graph structure
        for all nodeid m ∈ N.AdjacencyList do
            Emit(nid m, p)                          ▷ Pass PageRank mass to neighbors

class Reducer
    method Reduce(nid m, [p1, p2, ...])
        M ← ∅
        for all p ∈ counts [p1, p2, ...] do
            if IsNode(p) then
                M ← p                               ▷ Recover graph structure
            else
                s ← s + p                           ▷ Sum incoming PageRank contributions
        M.PageRank ← s
        Emit(nid m, node M)
```


PageRank: Example



PageRank in MapReduce

- Sketch of the MapReduce algorithm
  - The algorithm maps over the nodes
  - For each node computes the PageRank mass the needs to be distributed to neighbors
  - Each fraction of the PageRank mass is emitted as the value, keyed by the node ids of the neighbors
  - In the shuffle and sort, values are grouped by node id
    - Also, we pass the graph structure from mappers to reducers (for subsequent iterations to take place over the updated graph)
  - The reducer updates the value of the PageRank of every single node

PageRank in MapReduce

- Implementation details
  - Loss of PageRank mass for sink nodes
  - Auxiliary state information
  - One iteration of the algorithm
    - Two MapReduce jobs: one to distribute the PageRank mass, the other for dangling nodes and random jumps
  - Checking for convergence
    - Requires a driver program
    - When updates of PageRank are “stable” the algorithm stops

Graph Analysis Beyond MapReduce

Acknowledgements

- *Credits*
  - *Part of the course material is based on slides provided by the following authors*
    - *Grzegorz Malewicz, Matthew Austern, Aart Bik, James Dehnert, Ilan Horn, Naty Leiser, Grzegorz Czajkowski (Google, Inc.)*

Pregel: A System for Large-Scale Graph Processing

- What is it?
  - Model for fault-tolerant parallel processing of graphs
  - C++ API allowing users to apply this model
- Why use it?
  - Problems solvable with graph algorithms are common
  - The alternatives aren't very good
    - Develop distributed architecture for individual algorithms
    - Existing distributed platform (e.g., MapReduce)
      - May not be very good at graph algorithms (multiple stages → lots of overhead)

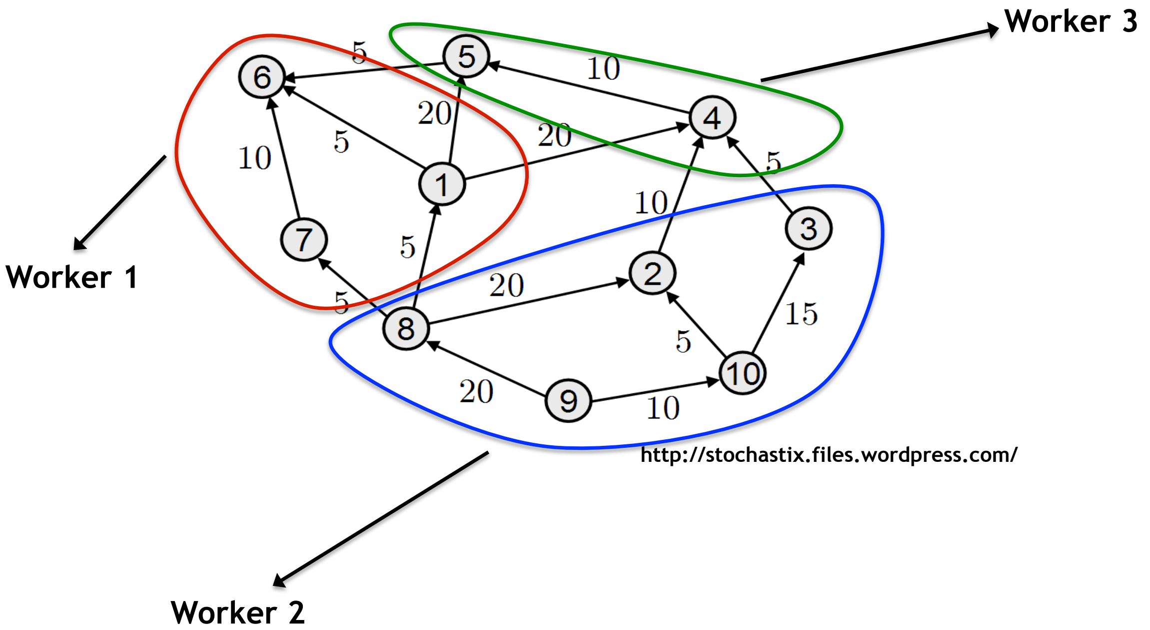
The Pregel model (1/2)

- Master/Worker model
  - Each worker assigned a subset of a directed graph's vertices
- Vertex-centric model. Each vertex has:
  - An arbitrary "value" that can be get/set.
  - List of messages sent to it
  - List of outgoing edges (edges have a value too)
  - A binary state (active/inactive)

The Pregel model (2/2)

- Bulk Synchronous Parallel model
  - Synchronous iterations of asynchronous computation
  - Master initiates each iteration (called a "superstep")
  - At every superstep
    - Workers asynchronously execute a user function on all of its vertices
    - Vertices can receive messages sent to it in the last superstep
    - Vertices can send messages to other vertices to be received in the next superstep
    - Vertices can modify their value, modify values of edges, change the topology of the graph (add/remove vertices or edges)
    - Vertices can "vote to halt"
  - Execution stops when all vertices have voted to halt and no vertices have messages.
  - Vote to halt trumped by non-empty message queue

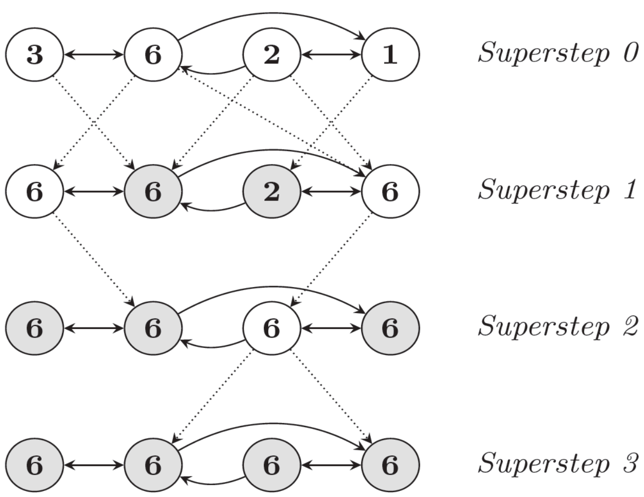
Illustration: vertex partitions



Loading the graph input

- Master assigns section of input to each worker
- Vertex "ownership" determined by hash(v) mod N
  - N - number of partitions
  - Recall each worker is assigned one or more partitions
  - User can modify this to exploit data locality
- Worker reads its section of input:
  - Stores vertices belonging to it
  - Sends other vertices to the appropriate worker
- Input stored on something like GFS
  - Section assignments determined by data locality

Simple example: find max

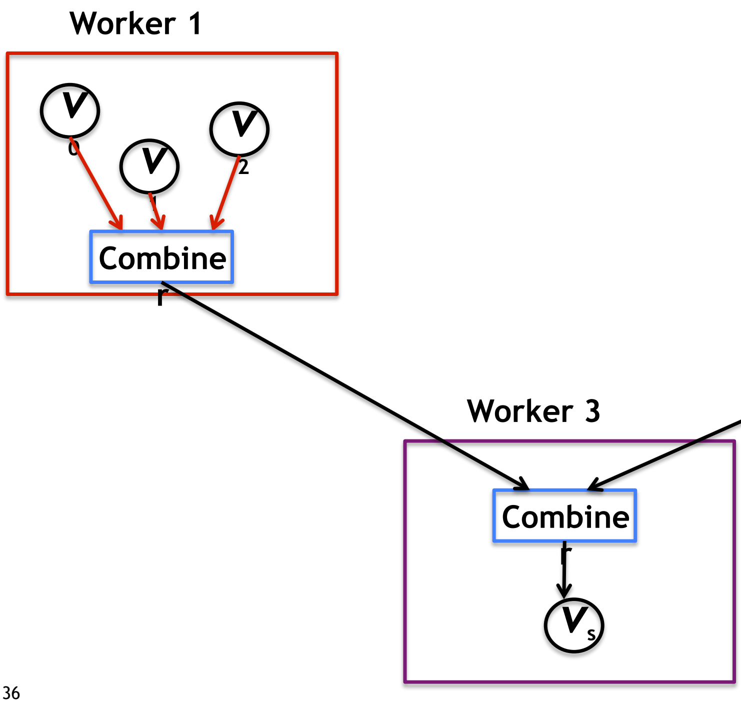



```
i_val := val
for each message m
if m > val then val := m
if i_val == val then
vote_to_halt
else
for each neighbor v
send_message(v, val)
```

Combiners

- ❑ Sometimes vertices only care about a summary value for the messages it is sent (e.g., previous example)
- ❑ Combiners allow for this (examples: min, max, sum, avg)
- ❑ Messages combined locally and remotely
- ❑ Reduces bandwidth overhead
- ❑ User-defined, not enabled by default

Combiners



Fault Tolerance (1/2)

- At start of superstep, master tells workers to save their state:
  - Vertex values, edge values, incoming messages
  - Saved to persistent storage
- Master saves aggregator values (if any)
- This isn't necessarily done at every superstep
  - That could be very costly
  - Authors determine checkpoint frequency using mean time to failure model

Fault Tolerance (2/2)

- When master detects one or more worker failures:
  - All workers revert to last checkpoint
  - Continue from there
  - That's a lot of repeated work!
  - At least it's better than redoing the whole thing.

Example: PageRank

```
class PageRankVertex
    : public Vertex<double, void, double> {
 public:
  virtual void Compute(MessageIterator* msgs) {
    if (superstep() >= 1) {
      double sum = 0;
      for (; !msgs->Done(); msgs->Next())
        sum += msgs->Value();
      *MutableValue() =
          0.15 / NumVertices() + 0.85 * sum;
    }

    if (superstep() < 30) {
      const int64 n = GetOutEdgeIterator().size();
      SendMessageToAllNeighbors(GetValue() / n);
    } else {
      VoteToHalt();
    }
  }
};
```

$$PR(p_i; t+1) = \frac{1-d}{N} + d \sum_{p_j \in M(p_i)} \frac{PR(p_j; t)}{L(p_j)}$$

http://wikipedia.org

Alternatives to Pregel

- Pregel is a Google project
- Alternative open source project similar in spirit
  - GPS: A Graph Processing System
    - Developed at Stanford Univ.
  - Giraph
    - An Apache project
- Other alternatives, tailored to the specific problem
  - E.g. “Filtering: A Method for Solving Graph Problems in MapReduce”