$$\alpha \qquad \dfrac{\begin{matrix}[\alpha]\\ \mathcal{D}\\ \beta\end{matrix}}{\alpha \to \beta} \qquad \dfrac{\begin{matrix}\mathcal{D}_1 \\ \alpha\end{matrix} \quad \begin{matrix}\mathcal{D}_2 \\ \alpha \to \beta\end{matrix}}{\beta} \qquad h(\mathcal{D}) = ?$$

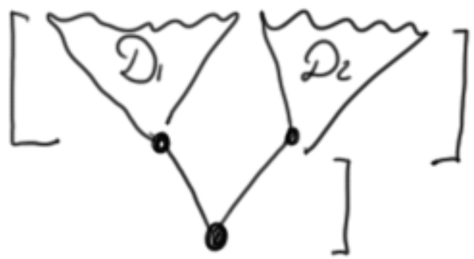
$$h(\alpha) = 0$$

$$h\left(\dfrac{\begin{matrix}\mathcal{D}_1 \\ \alpha\end{matrix} \quad \begin{matrix}\mathcal{D}_2 \\ \alpha \to \beta\end{matrix}}{\beta}\right) = \max\{h(\mathcal{D}_1), h(\mathcal{D}_2)\} + 1$$

$$h\left(\dfrac{\begin{matrix}[\alpha]\\ \mathcal{D}\\ \beta\end{matrix}}{\alpha \to \beta}\right) = h(\mathcal{D}) + 1$$

$$h(\alpha) = 0$$

$$h\left(\frac{\begin{matrix}\mathcal{D}_1 & \mathcal{D}_2 \\ \alpha & \alpha \to \beta\end{matrix}}{\beta}\right) = \max\{h(\mathcal{D}_1), h(\mathcal{D}_2)\} + 1$$



$$h\left(\frac{\begin{matrix}[\alpha] \\ \mathcal{D} \\ \beta\end{matrix}}{\alpha \to \beta}\right) = h(\mathcal{D}) + 1$$

$$\underbrace{\Gamma \vDash \phi} \;\&\; \underbrace{\Gamma' \vDash \phi'} \Rightarrow \Gamma, \Gamma' \vDash \phi \wedge \phi'$$

$$\forall v \left([\![\Gamma]\!]_v = 1 \Rightarrow [\![\varphi]\!]_v = 1 \right) \;\&\; \forall v \left([\![\Gamma']\!]_v = 1 \Rightarrow [\![\varphi']\!]_v = 1 \right)$$

$$\Downarrow$$

$$\forall v \Big(\big(\underbrace{[\![\Gamma]\!]_v = 1 \;\&\; [\![\Gamma']\!]_v = 1} \big) \Rightarrow \big(\underbrace{[\![\varphi]\!]_v = 1 \;\&\; [\![\varphi']\!]_v = 1} \big) \Big)$$

$$\forall v \left([\![\Gamma, \Gamma']\!]_v = 1 \Rightarrow [\![\varphi \wedge \varphi']\!]_v = 1 \right)$$

$$\Gamma, \Gamma' \vDash \varphi \wedge \varphi'$$

$$\Gamma \models \bot \Rightarrow \Gamma - \{\neg\phi\} \models \phi$$

$P \Rightarrow \varphi$

NON P OR φ

$\neg\phi \notin \Delta$

$$\underbrace{\Delta, \neg\varphi} \models \bot \;\Rightarrow\; \Delta \models \varphi$$

$$\forall v \left(\left([\![\Delta]\!]_v = 1 \;\&\; [\![\neg\varphi]\!]_v = 1 \right) \Rightarrow [\![\bot]\!]_v = 1 \right)$$

$\Leftrightarrow$

$$\forall v \left([\![\Delta]\!]_v \neq 1 \text{ OR } [\![\neg\varphi]\!]_v = 0 \text{ OR } \underbrace{[\![\bot]\!]_v = 1}_{\text{MAI}} \right)$$

$\Updownarrow$

$$\forall v \left([\![\Delta]\!]_v \neq 1 \text{ OR } [\![\neg\varphi]\!]_v = 0 \right)$$

$\Leftrightarrow$

$$\forall v \left([\![\Delta]\!]_v \neq 1 \text{ OR } [\![\varphi]\!]_v = 1 \right)$$

$\Leftrightarrow$

$$\forall v \left([\![\Delta]\!]_v = 1 \Rightarrow [\![\varphi]\!]_v = 1 \right)$$

$$\Gamma \vDash \sigma \;\&\; \Gamma', \sigma \vDash \phi \Rightarrow \Gamma, \Gamma' \vDash \phi$$

$$\forall v \left([\![\Gamma]\!]_v = 1 \Rightarrow [\![\sigma]\!]_v = 1 \right) \;\&\; \forall \sigma \left(\left([\![\Gamma']\!]_v = 1 \;\&\; [\![\sigma]\!]_v = 1 \right) \Rightarrow [\![\varphi]\!]_v = 1 \right)$$

$$[\![\Gamma]\!]_v = 1 \Rightarrow [\![\sigma]\!]_v = 1$$

$$P \Rightarrow Q$$

$$P \;\&\; Z \Rightarrow Q \;\&\; Z$$

$$\boxed{[\![\Gamma]\!]_v = 1 \;\&\; [\![\Gamma']\!]_v = 1} \Rightarrow$$

$$\Rightarrow [\![\sigma]\!]_v = 1 \;\&\; [\![\Gamma']\!]_v = 1$$

$$\Rightarrow \boxed{[\![\varphi]\!]_v = 1}$$

$\langle S, \sqsubseteq \rangle$ $\sqsubseteq \subseteq S \times S$ è RIFL, TRANS. ANTISIMM.

A $\langle \underbrace{\mathcal{P}(A), \subseteq}_{\text{ORDINE PARZIALE}} \rangle$ $\qquad x \in \mathcal{P}(A)$ $\quad x \subseteq A$

1) $\forall x \in \mathcal{P}(A) \quad x \subseteq x$

2) $\forall x \forall y \forall z \in \mathcal{P}(A) \; (\overline{x \subseteq} y \;\&\; y \subseteq z) \Rightarrow (x \subseteq z)$

→ 3) $\forall x \forall y \in \mathcal{P}(A) \; (x \subseteq y \;\&\; y \subseteq x) \Rightarrow x = y$ ←

x

$P \subseteq Q \Leftrightarrow \forall a \, (a \in P \Rightarrow a \in Q)$

$\left[\forall a \, (a \in X \Rightarrow a \in Y) \;\&\; \forall b \, (b \in Y \in b \in Z)\right]$ $\quad a \in X \Rightarrow a \in Y \Rightarrow a \in Z$

$\langle S, \sqsubseteq \rangle$

m è un elemento massimale se

$m \in S$ & $\forall y \in S \, (m \sqsubseteq y \Rightarrow m = y)$

$S \subseteq \wp(A)$ $\langle S, \subseteq \rangle$

M è massi

$\forall y \in S \, (m \subseteq y \Rightarrow m = y)$

$$\underbrace{\Gamma \vdash \bot} \Rightarrow \Gamma \vdash \alpha$$

$$\exists \begin{array}{c} \mathcal{D} \\ \bot \end{array} \quad hp(\mathcal{D}) \subseteq \Gamma$$

$$\left.\begin{array}{c} \mathcal{D} \\ \bot \\ \hline \alpha \end{array}\right\} \mathcal{D}' \qquad hp\Big(\begin{array}{c} \mathcal{D}' \\ \alpha \end{array}\Big) \subseteq \Gamma$$

$$\Leftrightarrow$$

$$\Gamma \vdash \alpha$$