

Esercizio 1 ii)

$$\begin{cases} y' = 1 + y^2 \\ y(0) = 1 \end{cases}$$

$$\begin{cases} y' = f(x)g(y) \\ y(x_0) = y_0 \end{cases}$$

$$\begin{aligned} f(x) &\equiv 1 \in C^0(\mathbb{R}) & I &= \mathbb{R} \\ g(y) &= 1 + y^2 \in C^0(\mathbb{R}) & J &= \mathbb{R} \end{aligned}$$

$$1 + y^2(x) \neq 0$$

$$y' = 1 + y^2 \iff \int \frac{dy}{1+y^2} = \int dx \quad \text{atn } y(x) = x + c, \quad c \in \mathbb{R}$$

$$\text{se } x+c \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \quad \text{ovvero } x \in \left(-\frac{\pi}{2}-c, \frac{\pi}{2}-c\right) \implies y(x) = \text{tg}(x+c)$$

$$c \text{ deve essere tale che } -\frac{\pi}{2} < x+c < \frac{\pi}{2}$$

$$\text{tg}(0+c) = 1 \quad \text{tg } c = 1 \implies c = \frac{\pi}{4}$$

$$y(x) = \text{tg}\left(x + \frac{\pi}{4}\right) \quad \forall x \in \left(-\frac{3}{4}\pi, \frac{\pi}{4}\right) = I'$$

Verifica

$$y(x) = \text{tg}\left(x + \frac{\pi}{4}\right)$$

$$y'(x) = 1 + \text{tg}^2\left(x + \frac{\pi}{4}\right) = 1 + [y(x)]^2$$

$$y(0) = \text{tg}\left(0 + \frac{\pi}{4}\right) = 1$$

NOTA Per la parte teorica si rimanda ai testi di Analisi Matematica