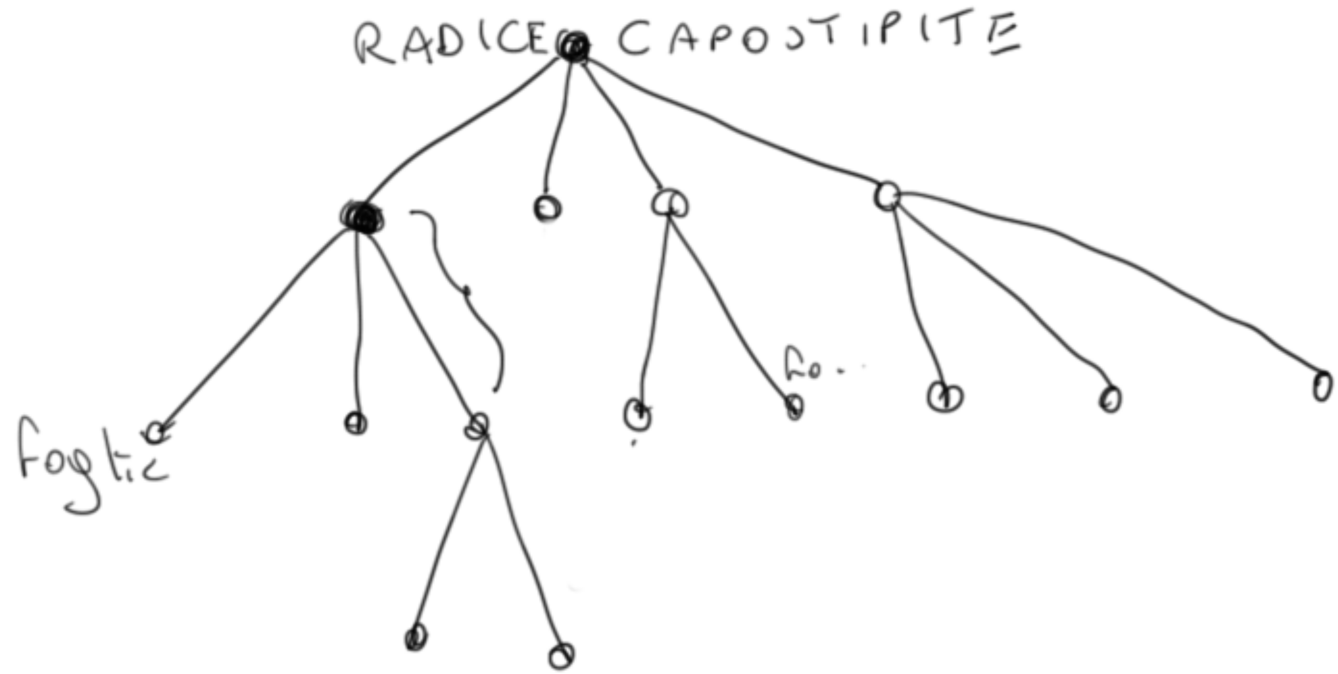

RADICE
CAPOSTIPITE
foglie
fo..

$c(N) < \aleph_0$

$R \subseteq N \times N$

aRb a è il padre di b

$R \circ R = \{(x,y) \mid \exists z \; xRz \; \& \; zRy\}$

$R^0 = id = \{(x,x) \mid x \in N\}$

$R^{i+1} = R \circ R^i$

$R^* = \bigcup_{i \in \mathbb{N}} R^i$

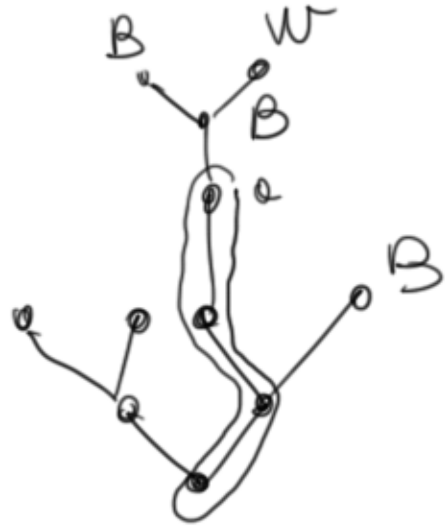
SI DICE CHE $\langle N, R \rangle$ è un albero

sse

1) $\langle N, R^* \rangle$ è un ordine parziale con minimo r (detto radice)

2) $\forall a \in N$ $\langle P_a, R^* \rangle$ è un ordine totale

dove $P_a = \{x \mid x R^* a\}$



B

W

1) $\vDash (\neg\alpha) \to (\alpha \to \bot)$

(2) $\vDash (\alpha \to \bot) \to (\neg\alpha)$

(1) $\forall v \quad [\![\neg\alpha]\!]_v = 0 \quad \text{OR} \quad [\![\alpha \to \bot]\!] = 1$

$$[\![\neg\alpha]\!]_v = 0 \iff [\![\alpha]\!]_v = 1 \qquad\qquad [\![\alpha \to \bot]\!] = 1 \iff [\![\alpha]\!]_v = 0$$

(2) $\forall v \quad [\![\alpha \to \bot]\!] = 0 \quad \text{OR} \quad [\![\neg\alpha]\!] = 1$

$$[\![\alpha \to \bot]\!] = 0 \iff [\![\alpha]\!]_v = 1 \qquad\qquad [\![\neg\alpha]\!] = 1 \iff [\![\alpha]\!]_v = 0$$

$$\forall_v \quad [\![(\alpha \to (\beta \to \gamma)) \to ((\alpha \wedge \beta) \to \gamma)]\!]_v = 1$$

$$[\![(\alpha \to (\beta \to \gamma)) \to ((\alpha \wedge \beta) \to \gamma)]\!]_v = 1$$

$$\Leftrightarrow$$

$$[\![\alpha \to (\beta \to \gamma)]\!]_v = 0 \quad \text{OR} \quad [\![(\alpha \wedge \beta) \to \gamma]\!]_v = 1$$

$$\Updownarrow \qquad\qquad\qquad \Updownarrow$$

$$[\![\alpha]\!]_v = 1 \;\&\; [\![\beta \to \gamma]\!] = 0 \qquad [\![\alpha \wedge \beta]\!]_v = 0 \quad \text{OR} \quad [\![\gamma]\!]_v = 1$$

$$\Updownarrow \qquad\qquad\qquad \Updownarrow$$

$$[\![\alpha]\!]_v = 1 \;\&\; [\![\beta]\!]_v = 1 \;\&\; [\![\gamma]\!]_v = 0 \qquad [\![\alpha]\!]_v = 0 \text{ OR } [\![\beta]\!]_v = 0 \text{ OR } [\![\gamma]\!]_v = 1$$

$\bot$

$\models \bot \to \alpha$

$\forall v \quad [\![\bot \to \alpha]\!]_v = 1$

$[\![\bot \to \alpha]\!]_v = 1 \iff \underbrace{[\![\bot]\!]_v = 0}_{\text{ok}}$ OR $[\![\alpha]\!]_v = 1$

$$\boxed{\dfrac{\bot}{\alpha}}$$

- SUPPONIAMO DI VOLER DIMOSTRARE α
MECCANISMO DI DIMOSTR. PER ASSURDO

1) SUPPONIAMO CHE α NON VALGA $(\neg \alpha)$

2) SOTTO L'ASSUNZIONE (1) ARRIVIAMO AD UN ASSURDO

$$\neg\alpha \equiv (\alpha \to \bot)$$

$$\dfrac{\begin{array}{c}[\neg\alpha] \\ \vdots \\ \bot\end{array}}{\alpha} \qquad\qquad \dfrac{\bot}{\alpha}$$

$\models \neg\neg\alpha \to \alpha \qquad \forall v \ [\![\neg\neg\alpha \to \alpha]\!]_v = 1$

$[\![\neg\neg\alpha \to \alpha]\!]_v = 1 \iff$

$[\![\neg\neg\alpha]\!]_v = 0$ OR $[\![\alpha]\!]_v = 1$

$\Updownarrow$

$[\![\neg\alpha]\!]_v = 1$

$\Updownarrow$

$[\![\alpha]\!]_v = 0$ — OR

$$\neg\neg\alpha \to \alpha$$

$$\begin{array}{c}[\neg\alpha]\\ \vert \\ \dfrac{\bot}{\alpha}\end{array} \qquad \dfrac{\bot}{\alpha}$$

$$\dfrac{\neg\beta \quad \beta}{\beta}$$

$$\dfrac{\beta\to\bot \quad \beta}{\bot}$$

$$\cfrac{\cfrac{\cfrac{[\neg\neg\alpha]^2 \quad [\neg\alpha]^1}{\bot}}{\alpha}\,1}{(\neg\neg\alpha)\to\alpha}\,2$$