Applicando il cambiamento di variabili

$$\iint_T xy\,dxdy = \iint_K \rho\cos\vartheta\,\rho\sin\vartheta\,\rho\,d\rho d\vartheta = \iint_K \rho^3 \cos\vartheta\sin\vartheta\,d\rho d\vartheta \underset{\sin 2\vartheta = 2\sin\vartheta\cos\vartheta}{=}$$

$$= \frac{1}{2}\iint_K \rho^3 \sin(2\vartheta)\,d\rho\,d\vartheta = \frac{1}{4}\iint_K \rho^3\, 2\sin(2\vartheta)\,d\rho\,d\vartheta =$$

$$= +\frac{1}{4}\int_0^1 d\rho \int_{\pi/4}^{\pi} \rho^3\, 2\sin(2\vartheta)\,d\vartheta = -\frac{1}{4}\left[\int_0^1 \rho^3 \left(\cos(2\vartheta)\Big|_{\frac{\pi}{4}}^{\pi}\right) d\rho\right] =$$

$$= -\frac{1}{4}\int_0^1 \rho^3 \Big(\underbrace{\cos(2\pi)}_{=1} - \underbrace{\cos\left(\frac{\pi}{2}\right)}_{=0}\Big)\,d\rho = -\frac{1}{4}\int_0^1 \rho^3\,d\rho = -\frac{1}{4}\,\frac{\rho^4}{4}\Big|_0^1 =$$

$$= -\frac{1}{16}$$

EX 3 foglio C parte 2