Applicando il cambiamento di variabile

$$\iint_T xy\,dx\,dy = \iint_K \rho\cos\vartheta\,\rho\sin\vartheta\,\underbrace{\rho}_{|J(\phi)|}\,d\rho\,d\vartheta = \iint_K \rho^3\cos\vartheta\sin\vartheta\,d\rho\,d\vartheta =$$

$\sin(2\vartheta) = 2\sin\vartheta\cos\vartheta$

$$= \frac{1}{2}\iint_K \rho^3\sin(2\vartheta)\,d\rho\,d\vartheta = \frac{1}{4}\iint_K \rho^3\, 2\sin(2\vartheta)\,d\rho\,d\vartheta =$$

$$= +\frac{1}{4}\int_0^1 d\rho \int_{\frac{5}{4}\pi}^{2\pi} \rho^3\, 2\sin(2\vartheta)\,d\vartheta = -\frac{1}{4}\left[\int_0^1 \rho^3 \left(\cos(2\vartheta)\Big|_{\frac{5}{4}\pi}^{2\pi}\right) d\rho\right] =$$

$$= -\frac{1}{4}\int_0^1 \rho^3 \left(\underbrace{\cos(4\pi)}_{=1} - \underbrace{\cos\left(\frac{5}{2}\pi\right)}_{=0}\right) d\rho = -\frac{1}{4}\int_0^1 \rho^3 d\rho = -\frac{1}{4}\frac{\rho^4}{4}\Big|_0^1 =$$

$$= -\frac{1}{16}$$

EX 3 foglio B parte 2