MATEMATICA
Università di Verona
Laurea in Biotecnologie - A.A. 2012/13
Lezione di martedì 11/12/2012

ESERCIZI DA FARE OGGI

Date le seguenti funzioni, studiarne l'andamento e provare a effettuare dei calcoli di aree, verificando l'attendibilità del risultato

$$\frac{x^2}{3x-1}, \quad \frac{3x+1}{x^2-x}, \quad \frac{\sin 2x}{5-\sin^2 x}, \quad \frac{\sqrt{1+\log x}}{x}, \quad \frac{x^3}{9-x^2}$$

Però prima volevo terminare di parlare dell'integrazione di funzioni razionali: ci manca l'ultimo caso $\boxed{\int \frac{P(x)}{x^2+px+q}\,dx}$ con $\Delta = p^2-4q < 0$

In questo caso si opera per "completamento di quadrati"

Ex. $\int \frac{x^3-x-1}{x^2+2x+5}\,dx$

1° passo: rendere il grado del numeratore < grado del denominatore

$$\begin{array}{l|l} x^3 \qquad\quad -x-1 & x^2+2x+5 \\ \hline -x^3-2x^2-5x & x-2 \ \text{quoziente} \\ // \ -2x^2-6x-1 & \\ \quad +2x^2+4x+10 & \\ \quad // \quad -2x+9 \ \leftarrow \text{resto} & \end{array}$$

$x^3-x-1 = (x^2+2x+5)(x-2) + (9-2x)$

$$= \int \left(x-2 - \frac{2x-9}{x^2+2x+5} \right) dx$$

$$= \int (x-2)\,dx - \int \frac{2x-9}{x^2+2x+5}\,dx$$

$$= \frac{x^2}{2} - 2x - \int \frac{2x-9}{x^2+2x+5}\,dx$$

mi resta da calcolare questo

$$\int \frac{2x-9}{x^2+2x+5}\,dx = \int \frac{2x-9}{(x^2+2x+1)+4}\,dx = \frac{1}{4}\int \frac{2x-9}{\left(\frac{x+1}{2}\right)^2+1}\,dx \quad \begin{bmatrix} \frac{x+1}{2} = t \\ x = 2t-1 \\ dx = 2\,dt \end{bmatrix} \quad = \frac{1}{\cancel{4}_2}\int \frac{4t-2-9}{t^2+1}\cdot\cancel{2}\,dt$$

$$= \frac{1}{2}\int \frac{4t-11}{t^2+1}\,dt = \frac{1}{2}\left(\int \frac{4t}{t^2+1}\,dt - 11\int \frac{1}{t^2+1}\,dt \right) = \frac{1}{2}\left(2\log(t^2+1) - 11\,\mathrm{arctg}(t) \right) + K$$

$$= \log\left(\left(\frac{x+1}{2}\right)^2+1 \right) - \frac{11}{2}\,\mathrm{arctg}\left(\frac{x+1}{2}\right) + K = \log\left(\frac{x^2+2x+5}{4}\right) - \frac{11}{2}\,\mathrm{arctg}\left(\frac{x+1}{2}\right) + K$$

$$= \log(x^2+2x+5) - \frac{11}{2}\,\mathrm{arctg}\left(\frac{x+1}{2}\right) + K.$$

Dunque $\int \frac{x^3-x-1}{x^2+2x+5}\,dx = \frac{x^2}{2} - 2x - \log(x^2+2x+5) + \frac{11}{2}\,\mathrm{arctg}\left(\frac{x+1}{2}\right) + K$. $\quad (K \in \mathbb{R})$

Ex. $\int \frac{x^4}{2x^2+3}\,dx = \frac{1}{2}\int \frac{x^4}{x^2+3/2}\,dx$

$$\begin{array}{l|l} x^4 & x^2+3/2 \\ \hline \qquad\qquad 9/4 & x^2-3/2 \end{array}$$

$x^4 = (x^2+3/2)(x^2-3/2) + 9/4$

$$= \frac{1}{2}\int \left(x^2 - \frac{3}{2} + \frac{9/4}{x^2+3/2} \right) dx = \frac{1}{2}\left(\frac{x^3}{3} - \frac{3}{2}x + \frac{9}{4}\int \frac{1}{x^2+3/2}\,dx \right)$$

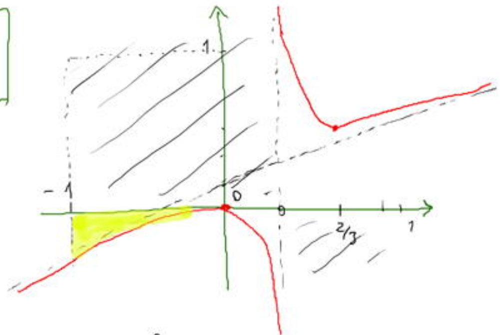
Vale $\int \frac{1}{x^2+3/2}\,dx = \frac{1}{3/2}\int \frac{1}{\frac{2}{3}x^2+1}\,dx = \frac{2}{3}\int \frac{1}{\left(\sqrt{2/3}\,x\right)^2+1}\,dx \quad \begin{bmatrix} x\sqrt{2/3} = t \\ x = \sqrt{3/2}\,t \\ dx = \sqrt{3/2}\,dt \end{bmatrix}$

$$= \frac{2}{3}\int \frac{1}{t^2+1}\cdot\sqrt{3/2}\,dt = \sqrt{2/3}\,\mathrm{arctg}\,t + K$$

$$= \sqrt{2/3}\,\mathrm{arctg}\left(x\sqrt{2/3}\right) + K$$

Dunque $\int \frac{x^4}{2x^2+3}\,dx = \frac{1}{2}\left(\frac{x^3}{3} - \frac{3}{2}x + \frac{9}{4}\sqrt{2/3}\,\mathrm{arctg}\left(x\sqrt{2/3}\right) \right) + K$.

$$\boxed{f(x) = \frac{x^2}{3x-1}}$$



Dominio: $x \neq 1/3$ $f(x)=0$ $x=0$ $f(x)>0$ $x>1/3$

Limiti $\lim_{x\to\pm\infty} f(x) = \pm\infty$ $\lim_{x\to 1/3^{\mp}} f(x) = \mp\infty$

Asintoti $\lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^2}{x(3x-1)} = \frac{1}{3} = m$

$\lim_{x\to\infty} \left(f(x) - \frac{1}{3}x\right) = \lim_{x\to\infty} \frac{\cancel{3x^2} - x(\cancel{3x}-1)}{3(3x-1)} = \frac{1}{9} = q$

$y = x/3 + 1/9$ Intersezioni? $f(x) = \frac{x}{3} + \frac{1}{9} = \frac{3x+1}{9}$

$9x^2 = (3x+1)(3x-1) = 9x^2 - 1$ NO

$f'(x) = \frac{2x(3x-1) - 3x^2}{(3x-1)^2} = \frac{x(3x-2)}{(3x-1)^2}$ $f'(x) = 0$ $x=0$ $f(0)=0$; $x = 2/3$ $f(2/3) = \frac{4/9}{1} = 4/9$

$f'(x) > 0$ $x<0 \lor x>2/3$

	0	1/3	2/3	
f'	+	−	−	+
f	↗ MAX	↘	↘ min	↗

$f''(x) = \frac{(6x-2)(3x-1)^{\cancel{2}} - 2\cdot 3\cancel{(3x-1)}(3x^2-2x)}{(3x-1)^{\cancel{4}3}} = 2\,\frac{\cancel{9x^2} - \cancel{6x} + 1 - \cancel{9x^2} + \cancel{6x}}{(3x-1)^3} = \frac{2}{(3x-1)^3}$

$f''(x) > 0$ per $x > 1/3$ f'': − | + (1/3) f: ∩ ∪

$\int_a^b f(x)\,dx$ per quali a, b? Bisogna che a e b stiano entrambi dalla stessa parte rispetto $1/3$ (cioè $a<1/3$, $b<1/3$ oppure $a>1/3$, $b>1/3$). Sarà $\int_a^b f(x)\,dx = F(b) - F(a)$ ove $F(x)$ è una primitiva di $f(x)$

$\int_{-1}^{0} \frac{x^2}{3x-1}\,dx = \begin{bmatrix} 3x-1 = t \\ x = \frac{t+1}{3} \\ dx = 1/3\,dt \end{bmatrix}$ $\int_{-4}^{-1} \frac{\left(\frac{t+1}{3}\right)^2}{t}\cdot\frac{1}{3}\,dt = \frac{1}{27}\int_{-4}^{-1} \frac{(t+1)^2}{t}\,dt = \frac{1}{27}\int_{-4}^{-1} \frac{t^2+2t+1}{t}\,dt = \frac{1}{27}\int_{-4}^{-1} \left(t+2+\frac{1}{t}\right)dt$

$= \frac{1}{27}\left[\frac{t^2}{2} + 2t + \log|t|\right]_{-4}^{-1} = \frac{1}{27}\left(\left(\frac{1}{2}-2+0\right) - (\cancel{8}-\cancel{8}+2\log 2)\right) = -\frac{1}{27}\left(\frac{3}{2} + 2\log 2\right)$ $\sim -1/9$

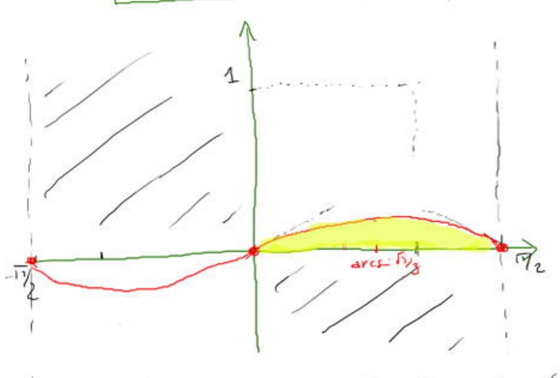
$$\boxed{f(x) = \frac{\sin 2x}{5 - \sin^2 x}}$$

Dominio: $\mathbb{R}$ Parità: $f(-x) = \frac{\sin(-2x)}{5-(\sin(-x))^2} = \frac{-\sin 2x}{5-\sin^2 x} = -f(x)$ dispari

Periodo: $\sin 2x$ ha periodo π

$\sin^2 x$: $\sin^2(x+\pi) = (\sin(x+\pi))^2 = (-\sin x)^2 = \sin^2 x$ ha periodo π

$\Rightarrow$ $f(x)$ ha periodo π. Ci conviene studiare $f(x)$ in $[-\pi/2, \pi/2]$ (dispari)



$f(-\pi/2) = f(\pi/2) = 0$ $f(0) = 0$ $f(x) = 0$ $\sin 2x = 0$ $2x = \pi + k\pi$

$x = -\pi/2, 0, \pi/2$ $f(x) > 0 \iff \sin 2x > 0 \iff 0 + 2k\pi < 2x < \pi + 2k\pi$

$\iff 0 + k\pi < x < \pi/2 + k\pi$

$f'(x) = \frac{2\cos 2x\,(5-\sin^2 x) - \sin 2x\,(-2\sin x\cos x)}{(5-\sin^2 x)^2}$

$= 2\,\frac{(1-2\sin^2 x)(5-\sin^2 x) + 2\sin^2 x\,(1-\sin^2 x)}{(5-\sin^2 x)^2} = 2\,\frac{-9\sin^2 x + 5}{(5-\sin^2 x)^2}$

$f'(x) = 0 \iff \sin^2 x = 5/9 \iff$ (per $0<x<\pi/2$) $\sin x = \sqrt{5}/3$ $x = \arcsin\sqrt{5}/3$ $\sim 43°$

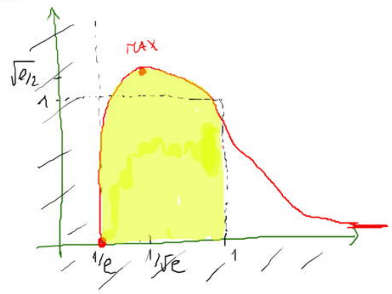
$f(\arcsin\sqrt{5}/3) = \frac{2\,\sqrt{5}/3\,\sqrt{1-5/9}}{5 - 5/9} = \sqrt{5}/10 \sim 0{,}22$ $f'(0) = \frac{2}{5}$ $f'(\pi/2) = -1/2$ $2\,\frac{-4}{\cancel{16}\,8}$

$\int_a^b f(x)\,dx$ ha senso per ogni a e b. Ad es.: $\int_0^{\pi/2} f(x)\,dx$

$\int_0^{\pi/2} \frac{\sin 2x}{5-\sin^2 x}\,dx = \int_0^{\pi/2} \frac{2\sin x\cos x}{5-\sin^2 x}\,dx$ $\begin{bmatrix} \sin x = t \\ \cos x\,dx = dt \end{bmatrix}$ $= \int_0^1 \frac{2t}{5-t^2}\,dt = \left[-\log|5-t^2|\right]_0^1$

$= (-\log 4) - (-\log 5) = \log 5 - \log 4 = \log 5/4$ $\sim 0{,}2$

$$\boxed{f(x) = \frac{\sqrt{1+\lg x}}{x}}$$



$\begin{cases} x>0 \\ \lg x \ge -1 \end{cases}$ $\quad x \ge 1/e$

$\lim_{x\to+\infty} f(x) \overset{H}{=} 0^+$

$f(1/e) = 0 \qquad f(x)>0 \quad \forall x > 1/e$

$$f'(x) = \frac{\frac{1/x}{2\sqrt{1+\lg x}}\cdot \cancel{x} - 1\cdot\sqrt{1+\lg x}}{x^2} = \frac{1-2(1+\lg x)}{2x^2\sqrt{1+\lg x}}$$

$$= -\frac{2\lg x+1}{2x^2\sqrt{1+\lg x}} \qquad f'(x)=0 \quad \lg x = -1/2 \quad x = \frac{1}{\sqrt{e}} \sim 0,5$$

$$f(1/\sqrt{e}) = \frac{\sqrt{1-1/2}}{1/\sqrt{e}} = \sqrt{\frac{e}{2}} \sim 1,2 \qquad f'(x)>0 \quad x<\frac{1}{\sqrt{e}}$$

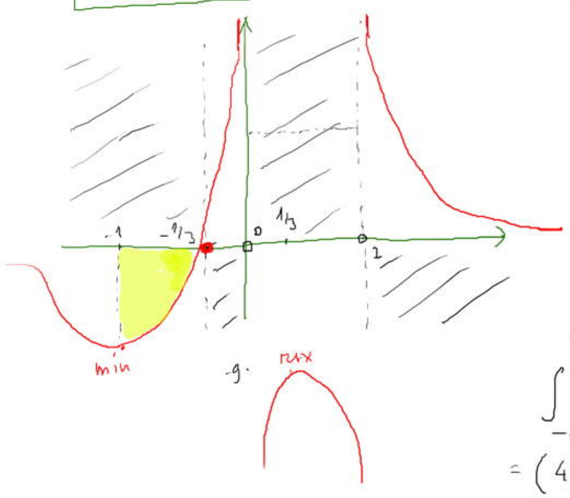
	$1/e$	$1/\sqrt{e}$	
f'	+		−
f	↗	max	↘

$$\lim_{x\to 1/e^+} f'(x) = -\infty$$

$$\int_{1/e}^{1} f(x)\,dx = \int_{1/e}^{1} \frac{\sqrt{1+\lg x}}{x}\,dx = \int_{1/e}^{1} \underbrace{\frac{1}{x}}_{\varphi'(x)} \underbrace{(1+\lg x)}_{\varphi(x)}^{\overset{\alpha}{1/2}}\,dx = \left(\frac{(1+\lg x)^{3/2}}{3/2}\right]_{1/e}^{1}$$

$$= \frac{2}{3} - 0 = 2/3$$

$$\boxed{f(x) = \frac{3x+1}{x^2-x}}$$



$x \ne 0, 1 \qquad f(x) = 0 \quad x = -1/3 \qquad f(x)>0 \qquad N(x)>0 \quad x>-1/3$

$D(x)>0 \quad x<0 \lor x>1$

	$-1/3$		0		1	
N	−		+		+	+
D	+		+		−	+
f	−		+		−	+

$\lim_{x\to-\infty} f(x) = 0^- \qquad \lim_{x\to+\infty} f(x) = 0^+ \qquad \lim_{x\to 0,1} f(x) = \infty$

$$f'(x) = \frac{3(x^2-x)-(3x+1)(2x-1)}{(x^2-x)^2} = \frac{3x^2-\cancel{3x}-6x^2+\cancel{3x}-2x+1}{(x^2-x)^2}$$

$$= -\frac{3x^2+2x-1}{(x^2-x)^2} \qquad f'(x)=0 \quad x = \frac{-1\pm\sqrt{1+3}}{3} = \left\langle \begin{matrix} -1 \\ 1/3 \end{matrix}\right.$$

$f'(x)>0$ per $-1<x<0$ e $0<x<1/3$

$f(-1) = -1 \qquad f(1/3) = -9$

	-1		0		$1/3$		1	
f'	−		+		+		−	−
f	↘	min	↗		↗	max	↘	↘

$\int_a^b f(x)\,dx$ con $\begin{matrix} a<0 \\ b<0 \end{matrix}$ oppure $\begin{matrix} 0<a<1 \\ 0<b<1 \end{matrix}$ oppure $\begin{matrix} a>1 \\ b>1 \end{matrix}$

$$\int_{-1}^{-1/3} \frac{3x+1}{x^2-x}\,dx = \int_{-1}^{-1/3} \left(\frac{4}{x-1} - \frac{1}{x}\right)dx$$

$$\frac{3x+1}{x^2-x} = \frac{a}{x-0} + \frac{b}{x-1} = \frac{(a+b)x-a}{x(x-1)}$$

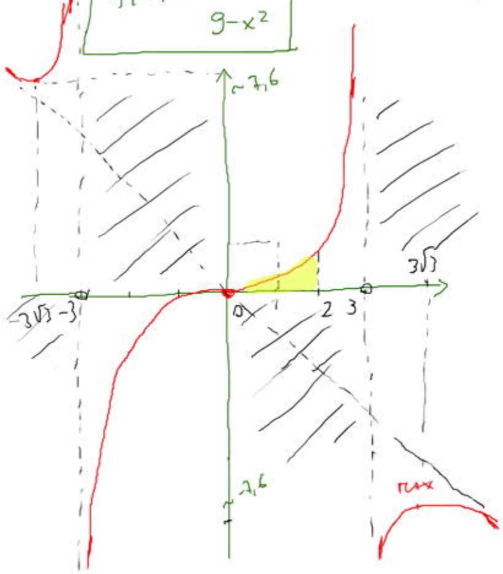
$$\begin{cases} a+b = 3 \\ -a = 1 \end{cases} \qquad \begin{cases} a=-1 \\ b=4 \end{cases}$$

$$= \left(4\lg|x-1| - \lg|x|\right]_{-1}^{-1/3}$$

$$= \left(4\lg 4/3 - \lg 1/3\right) - \left(4\lg 2 - \cancel{\lg 1}\right)$$

$$= 4(\lg 4 - \lg 3) + \lg 3 - 4\lg 2 = 8\lg 2 - 4\lg 3 + \lg 3 - 4\lg 2 = \underbrace{4\lg 2}_{\sim 2,8} - \underbrace{3\lg 3}_{\sim -3,3} \sim -0,5$$

$$\boxed{f(x) = \frac{x^3}{9-x^2}}$$



$x \neq \pm 3 \qquad f(-x) = \dfrac{(-x)^3}{9-(-x)^2} = \dfrac{-x^3}{9-x^2} = -f(x)$ dispari

$f(x) = 0 \quad x = 0 \qquad f(x) > 0 \quad N(x)>0 \;\; x>0 \quad D(x)>0 \;\; |x|<3$

	-3		0		3	
N	$-$	$-$		$+$		$+$
D	$-$	$+$		$+$		$-$
f	$+$	$-$		$+$		$-$

$\lim\limits_{x\to\pm\infty} f(x) = \mp\infty \qquad \lim\limits_{x\to\pm 3} f(x) = \infty$

Asintoto $\lim\limits_{x\to\infty} \dfrac{f(x)}{x} = -1 = m \qquad \lim\limits_{x\to\infty}(f(x) - mx) = \lim\limits_{x\to+\infty} \dfrac{\cancel{x^3}+9x-\cancel{x^3}}{9-x^2} = 0$

$y = -x$ a $\pm\infty$ Intersezioni? $\dfrac{x^{\cancel{3}\,2}}{9-x^2} = -\cancel{x}^{\,1} \qquad x = 0 \qquad x^2 = -9+x^2$ No

$$f'(x) = \frac{3x^2(9-x^2) - (-2x)\cdot x^3}{(9-x^2)^2} = \frac{x^2(27-3x^2+2x^2)}{(9-x^2)^2} = \frac{x^2(27-x^2)}{(9-x^2)^2}$$

$f'(x) = 0 \quad x = 0, \; x = \pm 3\sqrt{3} \qquad f(0) = 0 \qquad f(\pm 3\sqrt{3}) = \dfrac{\pm 81\sqrt{3}}{-18} = \mp \dfrac{9}{2}\sqrt{3} \sim 7{,}6$

$f'(x) > 0$ le $|x| < 3\sqrt{3}$

		$-3\sqrt{3}$		-3		0		3		$3\sqrt{3}$	
f'	$-$		$+$	0	$+$		$+$	0	$+$		$-$
f	$\searrow$		$\nearrow$		$\nearrow$		$\nearrow$		$\nearrow$		$\searrow$
		min				FLEX ORIZZ				max	

$x^3 = x(x^2-9) + 9x$

$\int_a^b f(x)\,dx$ vale le $\begin{matrix} a<-3 \\ b<-3 \end{matrix}$ oppure $\begin{matrix} -3<a<3 \\ -3<b<3 \end{matrix}$ oppure $\begin{matrix} a>3 \\ b>3 \end{matrix}$. $\displaystyle\int_0^2 \frac{x^3}{9-x^2}\,dx =$

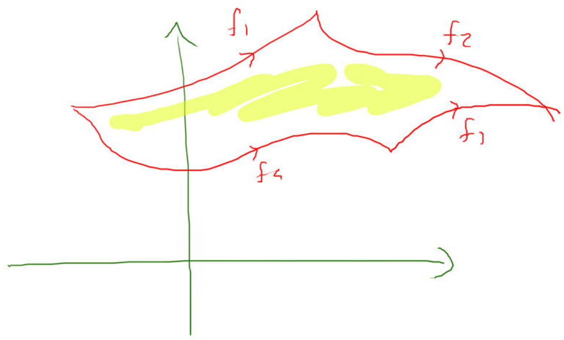
$$= -\int_0^2 \frac{x^3}{x^2-9}\,dx = -\int_0^2 \left(x + \frac{9x}{x^2-9}\right) dx = -\left(\frac{x^2}{2} + \frac{9}{2}\log|x^2-9|\right)\Big]_0^2 = -\left(\left(2+\frac{9}{2}\log 5\right) - \left(\frac{9}{2}\log 9\right)\right)$$

$$= -\left(2 + \frac{9}{2}\log\frac{5}{9}\right) = -\Big(2 + \frac{9}{2}(\underbrace{\log 5}_{\sim 1{,}5} - \underbrace{2\log 3}_{-2{,}2})\Big) = \frac{9}{2}(2\log 3 - \log 5) - 2 \sim 1{,}2$$

Esercizi per la prossima volta: Studiare le sgg. funzioni e calcolare l'area sottesa dai grafici per opportune scelte degli estremi.

$\dfrac{2x^2+5x-2}{x^2+x-2}$; $\dfrac{2x^2+5x+11}{x^2+4}$; $x^2 \log|2x+1|$; $\dfrac{x}{\sqrt{3-2x^2}}$; $\dfrac{1}{\sqrt{3-2x^2}}$ (arcsin x) ; $(2x+1)e^{3-x}$; $\dfrac{2+\sqrt{x}}{x+1}$

Inoltre vedremo:



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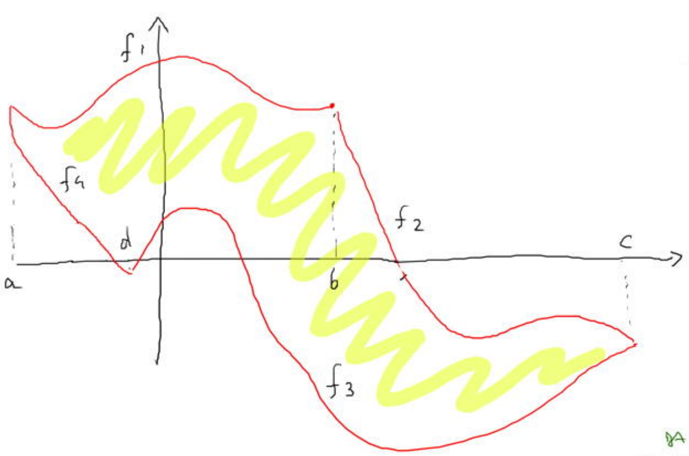
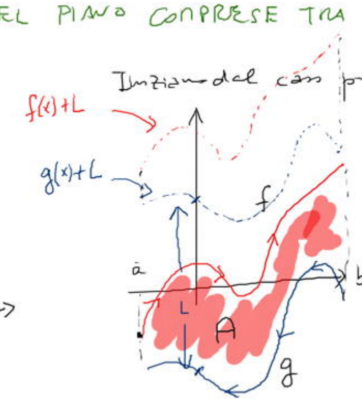
CALCOLO DI AREE DI ZONE LIMITATE DEL PIANO COMPRESE TRA GRAFICI DI FUNZIONI



Iniziamo dal caso più semplice: 2 funzioni



$$A = \int_a^b (f(x)+L)\,dx - \int_a^b (g(x)+L)\,dx$$

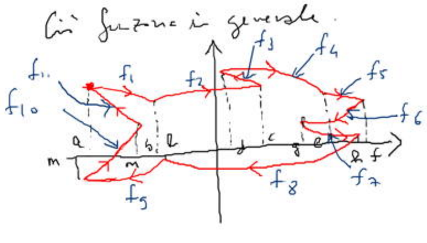
$$= \int_a^b f(x)\,dx + \cancel{\int_a^b L\,dx} - \int_a^b g(x)\,dx - \cancel{\int_a^b L\,dx}$$

$$= \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Scritta così, mi fa capire che percorro il grafico di f da a a b, quindi il grafico di g a ritroso da b ad a (e i tratti verticali non contano) →

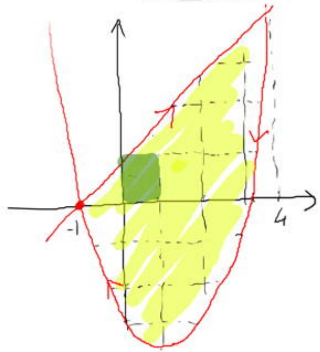
$$= \int_a^b f(x)\,dx + \int_b^a g(x)\,dx$$

Con funzioni in generale:



$$\text{Area} = \int_a^b f_1(x)\,dx + \int_b^c f_2(x)\,dx + \cdots + \int_l^m f_9(x)\,dx + \int_m^n f_{10}(x)\,dx + \int_n^a f_{11}(x)\,dx$$

Ex. $\boxed{\text{Disegnare e calcolare l'area di } S_1 = \{(x,y): x^2-2x-3 \le y \le x+1\}}$

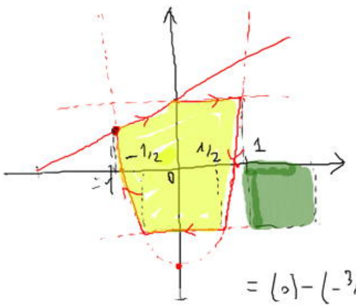


$$\text{Area} = \int_{-1}^{4} (x+1)\,dx + \int_{4}^{-1} (x^2-2x-3)\,dx = \left(\frac{x^2}{2}+x\right]_{-1}^{4} + \left(\frac{x^3}{3}-x^2-3x\right]_{4}^{-1}$$

$$= (12) - \left(-\frac{1}{2}\right) + \left(\frac{8}{3}\right) - \left(-\frac{20}{3}\right) = 12 + \frac{1}{2} + \frac{8}{3} + \frac{20}{3} = \frac{72+3+16+40}{6} = \frac{131}{6} \sim 22$$

$-1 \le y \le 1$

Ex. $S_2 = \{(x,y): 2x^2-3/2 \le y \le \frac{x}{2}+1,\ |y| \le 1\}$



$x/2+1 = 2x^2-3/2 \quad x+2 = 4x^2-3 \quad 4x^2-x-5=0 \quad x=-1 \quad (x=5)$

$1 = 2x^2-3/2 \quad 2 = 4x^2-3 \quad x = \pm 1$

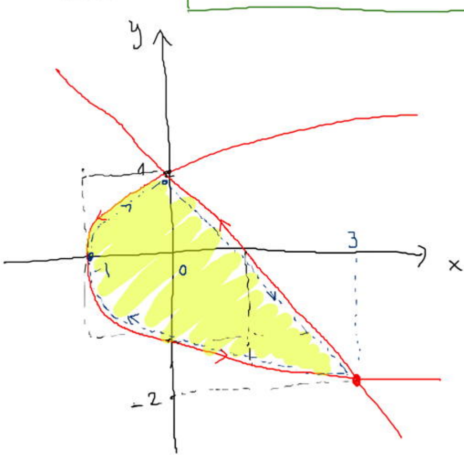
$-1 = 2x^2-3/2 \quad -2 = 4x^2-3 \quad x = \pm 1/2$

$$\text{Area} = \int_{-1}^{0} \left(\frac{x}{2}+1\right)dx + \int_{0}^{1} 1\,dx + \int_{1}^{1/2} (2x^2-3/2)\,dx + \int_{1/2}^{-1/2} (-1)\,dx + \int_{-1/2}^{-1} (2x^2-3/2)\,dx$$

$$= \left(\frac{x^2}{4}+x\right]_{-1}^{0} + \left(x\right]_{0}^{1} + \left(\frac{2x^3}{3}-\frac{3}{2}x\right]_{1}^{1/2} + \left(-x\right]_{1/2}^{-1/2} + \left(\frac{2x^3}{3}-\frac{3}{2}x\right]_{-1/2}^{-1} =$$

$$= (0)-\left(-\frac{3}{4}\right)+(1)-(0)+\left(\frac{1}{12}-\frac{3}{4}\right)-\left(-\frac{5}{6}\right)+\left(\frac{1}{2}\right)-\left(-\frac{1}{2}\right)+\left(\frac{5}{6}\right)-\left(-\frac{1}{12}+\frac{3}{4}\right) = 0+\frac{3}{4}+1-\frac{2}{3}+\frac{5}{6}+\frac{1}{2}+\frac{1}{2}+\frac{5}{6}-\frac{2}{3} = \frac{37}{12}$$

Ex. $\boxed{S_3 = \{(x,y): y^2-1 \le x \le 1-y\}}$



$y^2-1 = 1-y \quad y^2+y-2=0 \quad y = 1,\ -2$

$$\text{Area} = \int_{-2}^{1} (1-y)\,dy + \int_{1}^{-2} (y^2-1)\,dy = \left(y-\frac{y^2}{2}\right]_{-2}^{1} + \left(\frac{y^3}{3}-y\right]_{1}^{-2}$$

$$= \left(\frac{1}{2}\right) - (-4) + \cancel{\left(-\frac{2}{3}\right)} - \cancel{\left(-\frac{2}{3}\right)} = \frac{9}{2}$$

oppure in funzione di x:

$$\text{Area} = \int_{-1}^{0} \sqrt{x+1}\,dx + \int_{0}^{3} (1-x)\,dx + \int_{3}^{-1} \left(-\sqrt{x+1}\right)dx$$

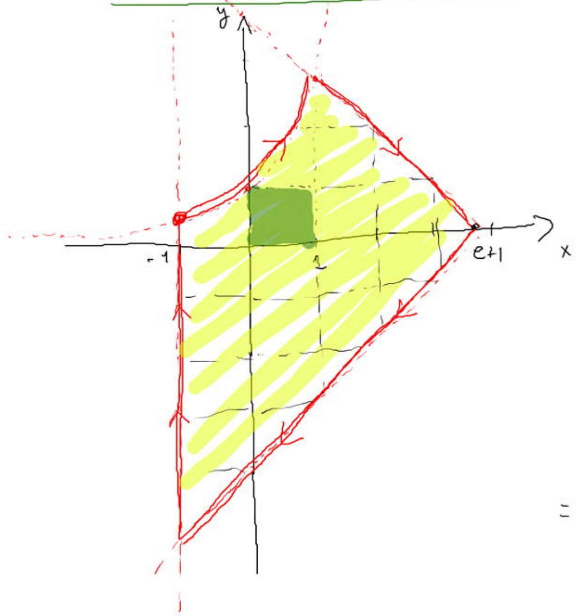
$$= \left(\frac{2}{3}(x+1)^{3/2}\right]_{-1}^{0} + \left(x-\frac{x^2}{2}\right]_{0}^{3} + \left(-\frac{2}{3}(x+1)^{3/2}\right]_{3}^{-1}$$

$$\int \sqrt{x+1}\,dx = \int (x+1)^{1/2}\,dx = \frac{(x+1)^{3/2}}{3/2} + k = \frac{2}{3}(x+1)\sqrt{x+1} + k$$

$$= \left(\frac{2}{3}\right) - (0) + \left(-\frac{3}{2}\right) - (0) + (0) - \left(-\frac{2}{3}\cdot 8\right)$$

$$= \frac{2}{3} - \frac{3}{2} + \frac{16}{3} = \frac{4-9+32}{6} = \frac{27}{6} = \frac{9}{2}$$

$$\boxed{S_4 = \{ (x,y) : \ y \le e^x , \ -1 \le x \le e+1-|y| \}}$$



$x \le e+1-|y| \qquad |y| \le e+1-x \quad \begin{cases} \nearrow (y>0) \quad y \le e+1-x \\ \searrow (y<0) \quad -y \le e+1-x \quad \text{cioè} \quad y \ge x-e-1 \end{cases}$

$e+1-x = e^x$ vero per $x=1$

$$\text{Area} = \int_{-1}^{1} e^x\,dx + \int_{1}^{e+1} (e+1-x)\,dx + \int_{e+1}^{-1} (x-e-1)\,dx$$

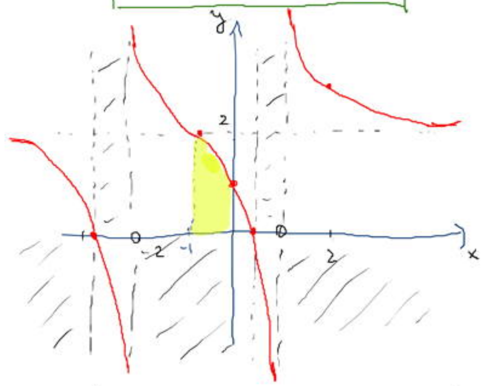
$$= \left(e^x\right]_{-1}^{1} + \left(\left(e+1\right)x - \tfrac{x^2}{2}\right]_{1}^{e+1} + \left(\tfrac{x^2}{2} - (e+1)x\right]_{e+1}^{-1}$$

$$= (e) - \left(\tfrac{1}{e}\right) + \left(\tfrac{(e+1)^2}{2}\right) - \left(e+\tfrac{1}{2}\right) + \left(e+\tfrac{3}{2}\right) - \left(-\tfrac{(e+1)^2}{2}\right)$$

$$= \cancel{e} - \frac{1}{e} + \frac{(e+1)^2}{2} - \cancel{e} - \frac{1}{2} + e + \frac{3}{2} + \frac{(e+1)^2}{2}$$

$$= \underline{(e+1)^2 + 1 + e - \tfrac{1}{e}} \sim 14{,}5 + 1 + 2{,}8 - 0{,}3 \sim 18{,}0$$

$$\boxed{f(x) = \frac{2x^2+5x-2}{x^2+x-2}}$$



$x \ne 1, -2 \qquad f(0) = 1 \qquad f(x) = 0 \quad x = \dfrac{-5 \pm \sqrt{25+16}}{4} = \dfrac{-5 \pm \sqrt{41}}{4}$

$\lim_{x\to\infty} f(x) = 2 \qquad f(x) = 2 \quad 2x^2+5x-2 = 2x^2+2x-4 \quad x = -2/3$

$f(x) > 0 \quad N(x) > 0 \ \ x < \dfrac{-5-\sqrt{41}}{4} \ \vee \ x > \dfrac{-5+\sqrt{41}}{4} \qquad D(x) > 0 \quad x < -2 \ \vee \ x > 1$

	$\frac{-5-\sqrt{41}}{4}$		-2		$\frac{-5+\sqrt{41}}{4}$		1	
N	+		−	−		+		+
D	+		+	−		−		+
f	+		−	+		−		+

$f(2) = \dfrac{12}{4} = 3$

$\lim_{x\to -2} f(x) = \infty \qquad \lim_{x\to 1} f(x) = \infty$

$$f'(x) = \frac{(4x+5)(x^2+x-2) - (2x+1)(2x^2+5x-2)}{(x^2+x-2)^2} = \frac{\cancel{4x^3} + 4x^2 - 8x + 5x^2 + 5x - 10 - \cancel{4x^3} - 10x^2 + 4x - 2x^2 - 5x + 2}{(x^2+x-2)^2}$$

$$= \frac{-3x^2-6x-10}{(x^2+x-2)^2} < 0 \ \text{sempre}$$

$\int_a^b f(x)$ ha senso o per $\begin{vmatrix} a<-2 \\ b<-2 \end{vmatrix}$ o per $\begin{vmatrix} -2<a<1 \\ -2<b<1 \end{vmatrix}$ o per $\begin{vmatrix} a>1 \\ b>1 \end{vmatrix}$. Es.: $\int_{-1}^{0} f(x)\,dx$

$\int \dfrac{2x^2+5x-2}{x^2+x-2}\,dx \qquad \left(2x^2+5x-2 = 2(x^2+x-2) + (3x+2)\right)$

$= \int \left(2 + \dfrac{3x+2}{x^2+x-2}\right)dx = 2x + \int \dfrac{3x+2}{x^2+x-2}\,dx$ resta da calcolare questo

$\dfrac{3x+2}{x^2+x-2} = \dfrac{A}{x-1} + \dfrac{B}{x+2} = \dfrac{(A+B)x + (2A-B)}{(x-1)(x+2)}$

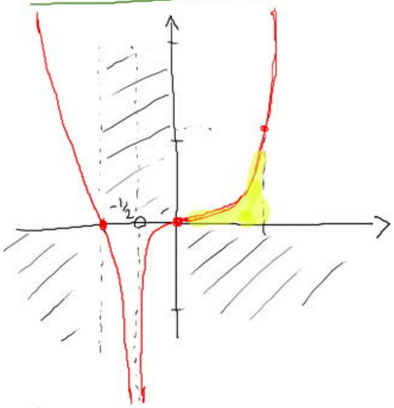
$\begin{cases} A+B = 3 \\ 2A-B = 2 \end{cases} \quad B = 3-A \quad 2A-3+A = 2 \quad A = 5/3 \quad B = 4/3$

Dunque $\int \dfrac{3x+2}{x^2+x-2} = \int \dfrac{5/3}{x-1}\,dx + \int \dfrac{4/3}{x+2}\,dx = \frac{5}{3}\lg|x-1| + \frac{4}{3}\lg|x+2|$

Quindi $\int f(x)\,dx = 2x + \frac{5}{3}\lg|x-1| + \frac{4}{3}\lg|x+2| + k$ dunque $\int_{-1}^{0} f(x)\,dx = \left[2x + \frac{5}{3}\lg|x-1| + \frac{4}{3}\lg|x+2|\right]_{-1}^{0} = \frac{4}{3}\lg 2 - \left(-2 + \frac{5}{3}\lg 2\right)$

$= 2 - \frac{1}{3}\lg 2 \sim 1{,}8$

$$f(x) = x^2 \log|2x+1|$$



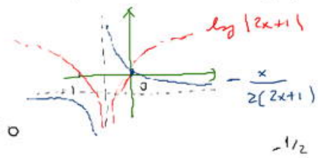
$x \neq -\frac{1}{2}$ $\quad f(x)=0 \quad x=0 \quad |2x+1|=1$ $\begin{cases} 2x+1=1 & x=0 \\ 2x+1=-1 & x=-1 \end{cases}$

$f(x)>0 \quad \log|2x+1|>0 \quad |2x+1|>1$ $\begin{cases} 2x+1>1 & x>0 \\ 2x+1<-1 & x<-1 \end{cases}$

$\lim_{x\to\pm\infty} f(x) = +\infty \qquad \lim_{x\to -1/2} f(x) = -\infty$

$$f'(x) = 2x\log|2x+1| + x^2\cdot\frac{1}{2x+1} = \underbrace{x}_{F_1}\overbrace{\left(2\log|2x+1| + \frac{x}{2x+1}\right)}^{F_2}$$

$f'(x)=0$ $\begin{cases} x=0 \\ \log|2x+1| = -\frac{x}{2(2x+1)} \end{cases}$



L'unico punto stazionario è $x=0$

$f'(x)>0$ $\quad F_1>0 \quad x>0$

$F_2>0 \quad \log|2x+1| > -\frac{x}{2(2x+1)} \quad x<-\frac12 \ \vee \ x>0$

	$-1/2$	0	
F_1	$-$	$-$	$+$
F_2	$+$	$-$	$+$
f'	$-$	$+$	$+$
f	$\searrow$	$\nearrow$	$\nearrow$

FLESSO ORIZ.

RESTA QUESTO

$\int_a^b f(x)\,dx$ ha senso se $\begin{cases} a<-1/2 \\ b<-1/2 \end{cases}$ oppure $\begin{cases} a>-1/2 \\ b>-1/2 \end{cases}$ $\quad f = \frac{2}{2x+1} \quad G = x^3/3$

Es. calcolare $\int_0^1 f(x)\,dx$

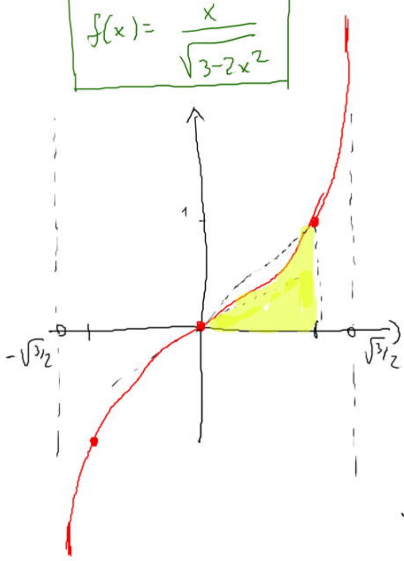
$$\int \underbrace{x^2}_{g} \underbrace{\log|2x+1|}_{F}\,dx = \frac{x^3}{3}\log|2x+1| - \int \frac{2}{2x+1}\cdot\frac{x^3}{3}\,dx = \frac{x^3}{3}\log|2x+1| - \frac{2}{3}\int\frac{x^3}{2x+1}\,dx$$

$$\int\frac{x^3}{2x+1}\,dx \quad \left(\begin{matrix} 2x+1=t \\ x=\frac{t-1}{2} \end{matrix} \quad dx = \tfrac{1}{2}dt\right) \quad = \int\frac{\left(\frac{t-1}{2}\right)^3}{t}\cdot\frac12\,dt = \frac{1}{16}\int\frac{(t-1)^3}{t}\,dt = \frac{1}{16}\int\frac{t^3-3t^2+3t-1}{t}\,dt =$$

$= \frac{1}{16}\int\left(t^2-3t+3-\frac1t\right)dt = \frac{1}{16}\left(t^3/3 - 3t^2/2 + 3t - \log|t|\right)+k$ e poi si rimette $2x+1$ al posto di t.

Dunque $\int_0^1 f(x)\,dx = \left(\frac{x^3}{3}\log|2x+1| - \frac{1}{24}\left(\frac{(2x+1)^3}{3} - \frac{3(2x+1)^2}{2} + 3(2x+1) - \log|2x+1|\right)\right]_0^1 = \left(\frac13\log 3 - \frac{1}{24}\left(\frac92 - \log 3\right)\right) - \left(-\frac{1}{24}\left(\frac{11}{6}\right)\right) = \ldots$

$$f(x) = \frac{x}{\sqrt{3-2x^2}}$$



$3-2x^2>0 \quad |x|<\sqrt{3}/2 \qquad f$ è dispari $\quad f(-x) = -f(x) \quad f(0)=0 \quad f(1)=1$

$\lim_{x\to 1^-} f(x) = +\infty$

$$f'(x) = \frac{1\cdot\sqrt{3-2x^2} - x\cdot\frac{1}{2\sqrt{3-2x^2}}\cdot(-4x)}{3-2x^2} =$$

$$= \frac{\cancel{2}(3-2x^2) + \overset{2}{\cancel{4}}x^2}{\cancel{2}(3-2x^2)\sqrt{3-2x^2}} = \frac{3}{(3-2x^2)^{3/2}} > 0 \quad f \text{ strett. crescente}$$

$f'(0) = 3/3\sqrt{3} = \frac{1}{\sqrt3} = \sqrt3/3$

$\int_a^b f(x)\,dx$ si può fare quando $\begin{cases} |a|<\sqrt3/2 \\ |b|<\sqrt3/2 \end{cases}$ $\quad$ Ex. $\int_0^1 f(x)\,dx$

$$\int_0^1 \frac{x}{\sqrt{3-2x^2}}\,dx = \int_0^1 x(3-2x^2)^{-1/2}\,dx = -\frac14\int_0^1 \underbrace{(-4x)}_{\varphi'(x)}(\underbrace{3-2x^2}_{\varphi(x)})^{-1/2}\,dx$$

$$= \left(-\frac14\cdot\frac{(3-2x^2)^{1/2}}{1/2}\right|_0^1 = \left(-\frac{\sqrt{3-2x^2}}{2}\right]_0^1 = \left(-\frac12\right) - \left(-\frac{\sqrt3}{2}\right) = \frac{\sqrt3-1}{2} \sim 0{,}35$$