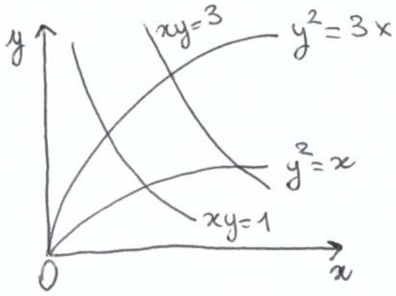
Esercizio 2



$$f(x,y)=xy \in C^0(\mathbb{R}^2) \Rightarrow f \in C^0(T)$$

f limitata, T chiuso e limitato, $f \in C^0(T) \Rightarrow$

$\Rightarrow$ f è integrabile su $T \Leftrightarrow \exists \iint_T f(x,y)\,dxdy$

Trasformo T in un rettangolo con una trasformazione che rettifichi le curve in figura

Assumo $\begin{cases} u = \dfrac{y^2}{x} \\ v = xy \end{cases}$

allora $\begin{cases} x = \dfrac{y^2}{u} \\ v = \dfrac{y^3}{u} \end{cases}$ $\begin{cases} x = y^2/u \\ y = \sqrt[3]{uv} \end{cases}$ $\begin{cases} x = \sqrt[3]{\dfrac{u^2v^2}{u^3}} \\ y = \sqrt[3]{uv} \end{cases}$ $\begin{cases} x = \sqrt[3]{\dfrac{v^2}{u}} = v^{\frac{2}{3}}u^{-\frac{1}{3}} \\ y = \sqrt[3]{uv} = u^{\frac{1}{3}}v^{\frac{1}{3}} \end{cases}$

$$J = \begin{bmatrix} -\dfrac{\sqrt[3]{v^2}}{3}u^{-\frac{4}{3}} & \dfrac{1}{3}u^{-\frac{2}{3}}v^{\frac{1}{3}} \\ +\dfrac{2}{3}v^{-\frac{1}{3}}u^{-\frac{1}{3}} & \dfrac{1}{3}u^{\frac{1}{3}}v^{-\frac{2}{3}} \end{bmatrix}$$

$$|J| = -\frac{1}{9}u^{-1} - \frac{2}{9}u^{-1} = -\frac{1}{3}\frac{1}{u}$$

T è il trasformato del quadrato $K = \{(x,y) \in \mathbb{R}^2 : u, v \in [1,3]\}$

Quindi

$$\iint_T xy\,dxdy = \iint_K -\frac{1}{3}\frac{1}{u}\sqrt[3]{\frac{v^2}{u}}\sqrt[3]{uv}\,dudv = \iint_K -\frac{1}{3}\frac{v}{u}\,dudv =$$

$$= -\frac{1}{3}\int_1^3 v\,dv \int_1^3 \frac{1}{u}\,du = -\frac{1}{3}\frac{v^2}{2}\Big|_1^3 \lg u\Big|_1^3 = -\frac{1}{3}\frac{1}{2}(9-1)(\lg 3 - \cancel{\lg 1}) =$$

$$= -\frac{4}{3}\lg 3.$$