Consideriamo

$x^2y^2 \geq 0$

$$0 \leq \frac{(xy)^2}{x^2+y^2} \leq \frac{1}{2}\,\frac{(x^4+y^4)}{x^2+y^2} \leq \frac{1}{2}\,\frac{x^4+2x^2y^2+y^4}{x^2+y^2} = \frac{1}{2}\,\frac{(x^2+y^2)^2}{x^2+y^2} = \frac{x^2+y^2}{2}$$

$0 \leq (x^2-y^2)^2 = x^4+y^4-2x^2y^2$

$x^2y^2 \leq \frac{1}{2}(x^4+y^4)$

$(x,y)\to(0,0)$: $\frac{(xy)^2}{x^2+y^2} \to 0$, $\quad \frac{x^2+y^2}{2} \to 0$

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{(xy)^2}{x^2+y^2} = 0$$

Quindi

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{\cos(xy)-1}{(xy)^2}\;\frac{(xy)^2}{x^2+y^2} = 0$$

EX foglio C parte 1