Esercizio 2 iii) fila A

$f(z) = \sin z$ è strettamente crescente in $[-\frac{\pi}{2}, \frac{\pi}{2}] \Rightarrow$ ivi f ammette inversa

$$\arcsin : [-1,1] \longrightarrow [-\tfrac{\pi}{2}, \tfrac{\pi}{2}]$$
$$x \longmapsto \arcsin x$$

$(\sin z)' = \cos z$ in $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\cos z \neq 0$ se $z \neq \pm\frac{\pi}{2} \iff x \neq \pm 1$

allora $\exists \ (\arcsin x)' \quad \forall x$ t.c. $x = \sin z \quad z \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ e $\cos z \neq 0$

sse $\forall x$ t.c. $x = \sin z$ e $z \in (-\frac{\pi}{2}, \frac{\pi}{2})$ sse $\forall x$ t.c. $x \in (-1,1)$.

$$(\arcsin x)' = \frac{1}{(\sin z)'}\Big|_{z=\arcsin x} = \frac{1}{(\cos z)}\Big|_{z=\arcsin x} = \frac{1}{\cos(\arcsin x)} =$$

in $(-\frac{\pi}{2}, \frac{\pi}{2})$ il coseno è positivo

$\cos^2 z = 1 - \sin^2 z$

$\cos z = \pm\sqrt{1-\sin^2 z}$

$$= \frac{1}{+\sqrt{1-(\sin(\arcsin x))^2}} = \frac{1}{+\sqrt{1-x^2}}, \quad \forall x \in (-1,1)$$

Esercizio 2 iii)

fila B

$f(\cdot) = \cos$ è strettamente decrescente in $[0,\pi] \Rightarrow \cos|_{[0,\pi]}$ ammette inversa

$$\arccos : [-1,1] \longrightarrow [0,\pi]$$
$$x \longmapsto \arccos x$$

$\exists \ (\arccos x)'$ nei punti x t.c. $x = \cos z$ con $(\cos z)' \neq 0$

$(\cos z)' = 0 \iff -\sin z = 0 \iff z = 0$ o $z = \pi$

quindi $\exists \ (\arccos x)' \quad \forall x$ t.c. $x = \cos z$ e $z \in (0,\pi) \iff \forall x$ t.c. $x \in (-1,1)$.

$$\forall x \in (-1,1) \quad (\arccos x)' = \frac{1}{(\cos z)'}\Big|_{z=\arccos x} = -\frac{1}{(\sin z)}\Big|_{z=\arccos x} =$$

$$= -\frac{1}{\sin(\arccos x)} = -\frac{1}{+\sqrt{1-(\cos(\arccos x))^2}} = -\frac{1}{\sqrt{1-x^2}}$$

seno è positivo