$$|\{a, b, c\}| = 3$$

$$|\mathbb{N}| = \aleph_0$$

$$|\mathbb{R}| = 2^{\aleph_0} = c$$

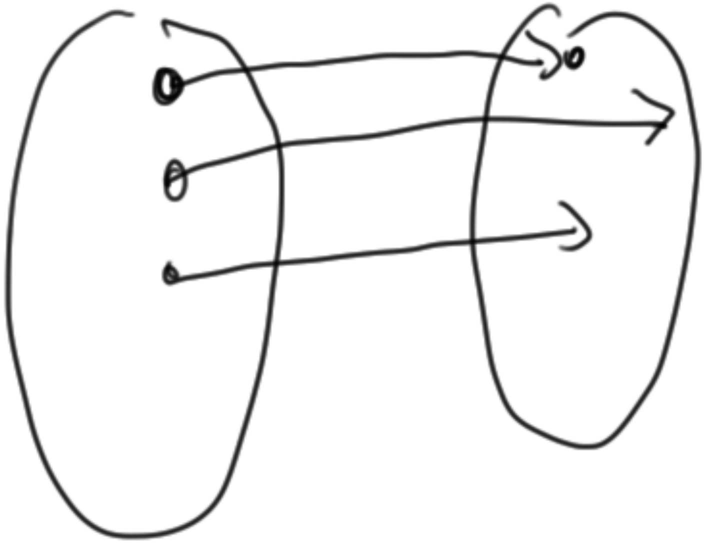
$$|\mathbb{Q}| = \aleph_0$$

$$\aleph_0 < 2^{\aleph_0}$$

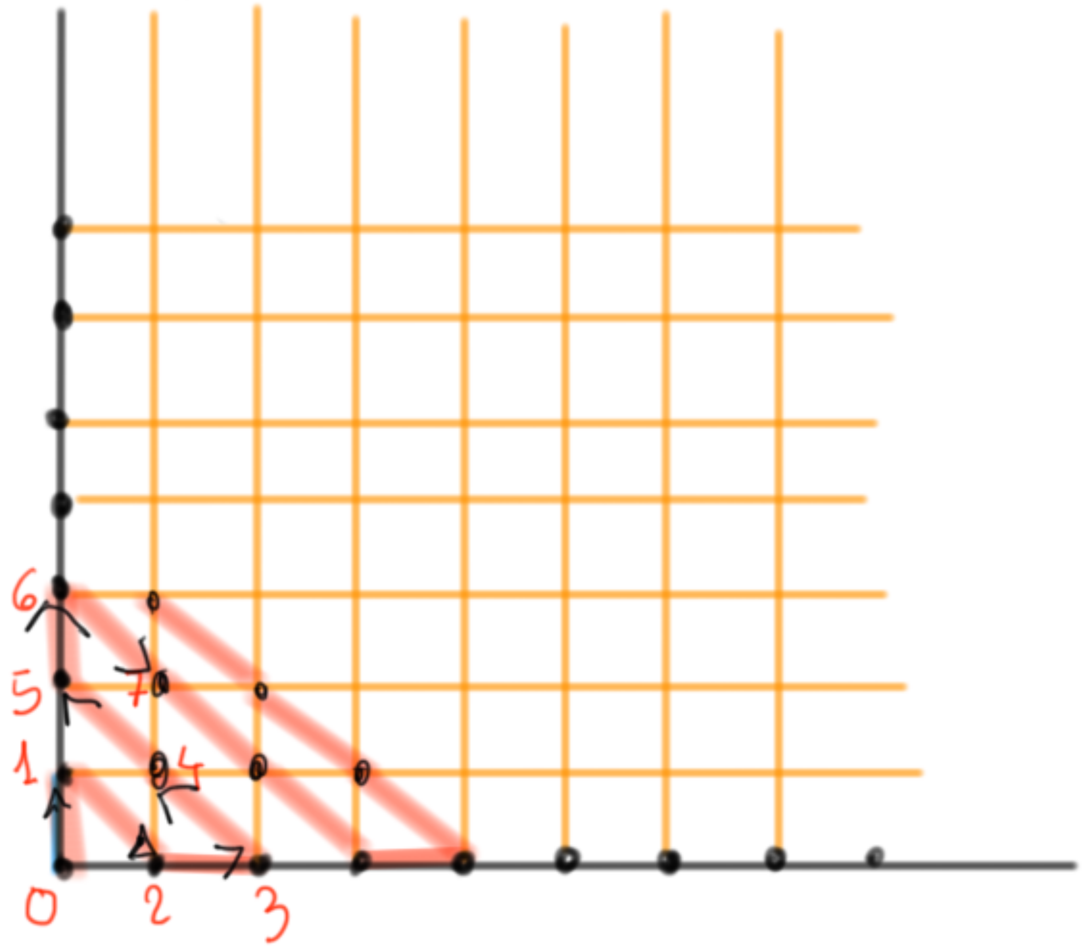
$A, B \qquad |A| = |B|$

$$\Longleftrightarrow$$

$\exists f : A \to B$ BIIETTIVA



$$|\mathbb{N}| = |\mathbb{N} \times \mathbb{N}|$$



$\mathbb{N}$

$$I_n = \{ x \mid x \in \mathbb{N} \ \& \ x < n \}$$

$$I_3 = \{0, 1, 2\}$$

$$I_2 = \{0, 1\}$$

$$I_1 = \{0\}$$

$$I_0 = \emptyset$$

A é finito se $\exists n$

t.c. $|A| = |I_n|$

$|I_n| = n$

$|\emptyset| = 0$ $|\emptyset| = |I_0| = 0$

$|A| \begin{matrix} \leq \\ = \\ \geq \end{matrix} |B|$

A finito $B \subseteq A$

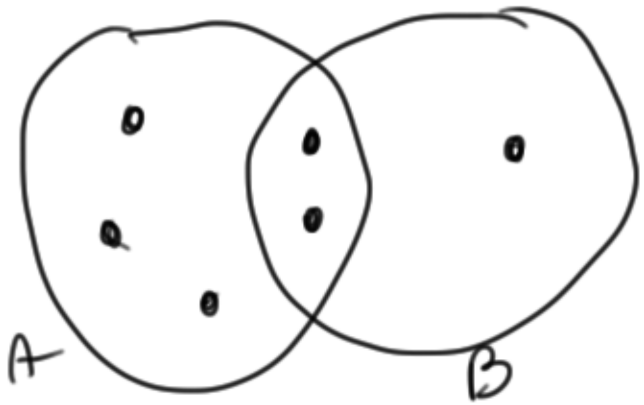
$$|B| \leq |A|$$

$$B \subseteq A \Rightarrow \begin{array}{l} |A| = I_n \\ |B| = I_m \end{array} \quad \& \quad m \leq n$$

A, B finiti

$A \cap B = \emptyset \Rightarrow |A \cup B| = |A| + |B|$

$|A \cup B| = |A| + |B| - |A \cap B|$



$|A| = 5$

$|B| = 3$

$|A \cup B| = 6 = 5 + 3 - 2$

A, B finiti

$$|A \times B| = |A| \cdot |B| \qquad |\{a\} \times B| = |B|$$

$$f(a,b) = b$$

$$A \times B = \bigcup_{a \in A} \{a\} \times B$$

$$a, b \in A, \quad a \neq b \Rightarrow \underbrace{\{a\} \times B} \cap \underbrace{\{b\} \times B} = \emptyset$$

$$|A \times B| = \sum_{a \in A} |\{a\} \times B| = \sum_{a \in A} |B| = \underbrace{|B| + \ldots + |B|}_{|A|}$$

$$= |A| \cdot |B|$$

A, B finiti

$$B^A \overset{\text{def}}{=} \{ f : A \to B \}$$

<u>TEO</u> $|B^A| = |B|^{|A|}$

$|A| = n$ $\quad A = \{a \ldots a_n\}$

$\varphi : B^A \to B^n \qquad \varphi(f) = (f(a_1), \ldots, f(a_n))$

$$|B^A| = |B^n| = |B|^n = |B|^{|A|} \text{ cvd}$$

$|C \times D| = |C| \times |D| \qquad |C^n| = |C|^n$

$A \quad A^n \quad A^0$

$$A^n = \{(a_0, \dots, a_{n-1}) \mid a_1, \dots, a_n \in A\}$$

$$A^n = \{f : I_n \to A\} \qquad n > 0$$

$$\{0, \dots, n-1\} \qquad I_0$$

$$|A^n| = |A^{I_n}| = |A|^{|I_n|} = |A|^n$$

$$|A^0| = |A^{I_0}| = |A|^{|I_0|} = |A|^0 = 1$$

A^0

$$A^{0} = \{ f \mid f : I_0 \to A \}$$

$$= \{ f \mid \underbrace{f : \emptyset \to A} \}$$

$f : \emptyset \to A$? ?

$A^{\emptyset} = \{ * \}$ $\quad * : \emptyset \to A$

$\downarrow$

$* = \emptyset$ $\quad \emptyset : \emptyset \to A$?

$\forall b \in \emptyset \ \exists c \in A \ (b,c) \in \emptyset$

$\forall b \, (b \in \emptyset \Rightarrow \exists c \ \ldots\ldots) \ V$

$g : B \to C$

1) $g \subseteq B \times C$

$\emptyset \subseteq \emptyset \times A$

2) $\forall b \in B \ \exists c \in C$
$(b,c) \in g$

3) (b,c)
$(b,c') \in g \Rightarrow c = c'$

A finito $\quad |\mathcal{P}(A)| = 2^{|A|}$

$$|\mathcal{P}(A)| = |\{0,1\}^A| = 2^{|A|}$$

$$\{0,1\}^A = \{ f \mid f : A \to \{0,1\} \}$$

$\psi : \mathcal{P}(A) \to \{0,1\}^A$ biiettiva.

$$f : A \to \{0,1\} \Rightarrow B_f = \{ a \mid f(a) = 1 \}$$

$B \subseteq A \qquad f_B : A \to \{0,1\} \qquad f_B(a) = 1 \iff a \in B$