

Considero

$$0 \leq \frac{(xy)^2}{x^2+y^2} \leq \frac{1}{2} \frac{x^4+y^4}{x^2+y^2} \leq \frac{1}{2} \frac{x^4+2x^2y^2+y^4}{x^2+y^2} = \frac{1}{2} \frac{(x^2+y^2)^2}{x^2+y^2} \leq \frac{x^2+y^2}{2}$$

↓  
0

$(x,y) \rightarrow (0,0)$

$$0 \leq (x^2-y^2)^2 \leq x^4+y^4-2x^2y^2$$
$$x^2y^2 \leq \frac{1}{2}(x^4+y^4)$$

↓  
0

$(x,y) \rightarrow (0,0)$

$$\Rightarrow \lim_{(x,y) \rightarrow (0,0)} \frac{(xy)^2}{x^2+y^2} = 0$$

Quindi

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y) = \lim_{(x,y) \rightarrow (0,0)} \frac{1-\cos(xy)}{(xy)^2} \cdot \frac{(xy)^2}{x^2+y^2} = 0$$