University of Verona Master's Program Mathematics

## Representation Theory Exam 2016/17

1. Determine the Auslander-Reiten-quiver of the path algebra  $\Lambda$  given by the quiver

$$1 \longrightarrow 2 \longleftarrow 3 \longrightarrow 4$$

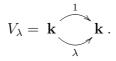
(hint: there are 10 indecomposables!)

Furthermore,

- (a) write down the almost split sequence starting at  $P_3$ .
- (b) Is there an indecomposable module M such that  $\dim \operatorname{Hom}(P_3, M) > 1$ ?
- (c) What is the maximal length of an indecomposable module?
- (d) Is there a non-split short exact sequence  $0 \to P_3 \to X \to I_4 \to 0$ ? (8 points)
- 2. Let **k** be a field and consider the path algebra  $\mathbf{k}\widetilde{\mathbb{A}_1}$ , where  $\widetilde{\mathbb{A}_1}$  is the quiver



Write down the representations of the indecomposable projective modules  $P_1$  and  $P_2$ . Moreover, for  $\lambda \in \mathbf{k}^*$ , consider the representation



At the level of representations, write down an injective map  $f: P_2 \to P_1$  such that Coker  $f = V_{\lambda}$ . (5 points)

- 3. Let  $\Lambda$  be a finite-dimensional algebra over an algebraically closed field.
  - (a) Give the definition of a left almost split morphism. (2 points)
  - (b) Let  $M, N \in \Lambda$ -mod and  $f: M \to N$  be left almost split. Show that M is indecomposable. (4 points)
- 4. Let R be a ring with Jacobson radical J, and let S be a simple left R-module.
  - (a) Show that there is a left ideal I of R such that  $S \cong R/I$ . (2 points)
  - (b) Deduce that  $J \cdot S = 0$ . (3 points)

(6 points)