University of Verona
Master's Program Mathematics

Representation Theory
Exam 2016/17

1. Determine the Auslander-Reiten-quiver of the path algebra Λ given by the quiver

$$1 \longrightarrow 2 \longleftarrow 3 \longrightarrow 4$$

(hint: there are 10 indecomposables!) *(6 points)*

Furthermore,

(a) write down the almost split sequence starting at P_3.

(b) Is there an indecomposable module M such that $\dim \operatorname{Hom}(P_3, M) > 1$?

(c) What is the maximal length of an indecomposable module?

(d) Is there a non-split short exact sequence $0 \to P_3 \to X \to I_4 \to 0$? *(8 points)*

2. Let $\mathbf{k}$ be a field and consider the path algebra $\mathbf{k}\widetilde{\mathbb{A}_1}$, where $\widetilde{\mathbb{A}_1}$ is the quiver

$$1 \rightrightarrows 2 \,.$$

Write down the representations of the indecomposable projective modules P_1 and P_2. Moreover, for $\lambda \in \mathbf{k}^*$, consider the representation

$$V_\lambda = \mathbf{k} \overset{1}{\underset{\lambda}{\rightrightarrows}} \mathbf{k} \,.$$

At the level of representations, write down an injective map $f\colon P_2 \to P_1$ such that $\operatorname{Coker} f = V_\lambda$. *(5 points)*

3. Let Λ be a finite-dimensional algebra over an algebraically closed field.

(a) Give the definition of a left almost split morphism. *(2 points)*

(b) Let $M, N \in \Lambda$-mod and $f\colon M \to N$ be left almost split. Show that M is indecomposable. *(4 points)*

4. Let R be a ring with Jacobson radical J, and let S be a simple left R-module.

(a) Show that there is a left ideal I of R such that $S \cong R/I$. *(2 points)*

(b) Deduce that $J \cdot S = 0$. *(3 points)*