

(submit your solutions during the exercise class on January 17, 2015)

1. Let  $k$  be an algebraically closed field and let

$$\Lambda = \begin{pmatrix} k & 0 & 0 \\ k & k & k \\ 0 & 0 & k \end{pmatrix}$$

be the algebra given by all matrices of the form  $\begin{pmatrix} k_1 & 0 & 0 \\ k_2 & k_3 & k_4 \\ 0 & 0 & k_5 \end{pmatrix}$  where  $k_1, \dots, k_5 \in k$ .

Find a quiver  $Q$  such that  $\Lambda$  is the path algebra of  $Q$ . (6 points)

2. Compute the Auslander-Reiten quiver of the path algebra given by the quiver

$$Q : \overset{1}{\bullet} \rightarrow \overset{2}{\bullet} \leftarrow \overset{3}{\bullet}.$$

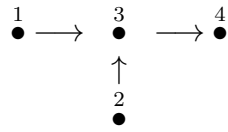
Use the Auslander-Reiten formula to compute  $\text{Ext}_{\Lambda}^1(I_i, P_1)$ ,  $i = 1, 2, 3$ . (10 points)

3. Compute the Auslander-Reiten quiver of the path algebra given by the quiver

$$\overset{1}{\bullet} \leftarrow \overset{2}{\bullet} \rightarrow \overset{3}{\bullet}$$

and determine the minimal projective resolution of the module  $I_2$ . (10 points)

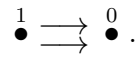
4. Compute the Auslander-Reiten quiver of the path algebra given by the quiver



and determine the representation  $\tau^-P_3$ .

(10 points)

5. Consider the Kronecker algebra, the path algebra given by the quiver



- (a) Determine the components  $\mathbf{p}$  and  $\mathbf{q}$  in the Auslander-Reiten quiver;
- (b) show that  $\mathbf{p}$  consists of the representations  $P_n, n \geq 0$ ,

$$K^n \begin{array}{c} \xrightarrow{A_n} \\ \xrightarrow{B_n} \end{array} K^{n+1}$$

where

$$A_n \cdot (x_1, \dots, x_n) = (x_1, \dots, x_n, 0)$$

$$B_n \cdot (x_1, \dots, x_n) = (0, x_1, \dots, x_n)$$

- (c) describe the representations in  $\mathbf{q}$ ;
- (d) prove that all modules  $X$  in  $\mathbf{p}$  satisfy  $\text{Ext}^1(X, X) = 0$ .

(12 points)

6. True or false? Consider the following statements over a finite dimensional algebra  $\Lambda$ .

- (a) The radical of any indecomposable injective  $\Lambda$ -module is superfluous.
- (b) The socle of any indecomposable projective  $\Lambda$ -module is superfluous.
- (c) The socle of any indecomposable projective  $\Lambda$ -module is indecomposable. (12 points)