Representation Theory - Exercises January 17, 2015

(**submit your solutions** during the excercise class on January 17, 2015)

1. Let k be an algebraically closed field and let

$$\Lambda = \begin{pmatrix} k & 0 & 0 \\ k & k & k \\ 0 & 0 & k \end{pmatrix}$$

be the algebra given by all matrices of the form $\begin{pmatrix} k_1 & 0 & 0 \\ k_2 & k_3 & k_4 \\ 0 & 0 & k_5 \end{pmatrix}$ where $k_1, \ldots, k_5 \in k$.

Find a quiver Q such that Λ is the path algebra of Q. (6 points)

2. Compute the Auslander-Reiten quiver of the path algebra given by the quiver

$$Q : \overset{1}{\bullet} \rightarrow \overset{2}{\bullet} \leftarrow \overset{3}{\bullet} \;.$$

Use the Auslander-Reiten formula to compute $\mathrm{Ext}^1_\Lambda(I_i, P_1)$, $i = 1, 2, 3$. (10 points)

3. Compute the Auslander-Reiten quiver of the path algebra given by the quiver

$$\overset{1}{\bullet} \leftarrow \overset{2}{\bullet} \rightarrow \overset{3}{\bullet}$$

and determine the minimal projective resolution of the module I_2. (10 points)

4. Compute the Auslander-Reiten quiver of the path algebra given by the quiver

$$\begin{array}{ccccc} \overset{1}{\bullet} & \longrightarrow & \overset{3}{\bullet} & \longrightarrow & \overset{4}{\bullet} \\ & & \uparrow & & \\ & & \overset{2}{\bullet} & & \end{array}$$

and determine the representation $\tau^{-}P_3$. (10 points)

5. Consider the Kronecker algebra, the path algebra given by the quiver

$$\overset{1}{\bullet} \rightrightarrows \overset{0}{\bullet}\,.$$

(a) Determine the components $\mathbf{p}$ and $\mathbf{q}$ in the Auslander-Reiten quiver;

(b) show that $\mathbf{p}$ consists of the representations $P_n, n \geq 0$,

$$K^n \underset{B_n}{\overset{A_n}{\rightrightarrows}} K^{n+1}$$

where

$$A_n \cdot (x_1, \ldots, x_n) = (x_1, \ldots, x_n, 0)$$
$$B_n \cdot (x_1, \ldots, x_n) = (0, x_1, \ldots, x_n)$$

(c) describe the representations in $\mathbf{q}$;

(d) prove that all modules X in $\mathbf{p}$ satisfy $\operatorname{Ext}^1(X, X) = 0$. (12 points)

6. True or false? Consider the following statements over a finite dimensional algebra Λ.

(a) The radical of any indecomposable injective Λ-module is superfluous.

(b) The socle of any indecomposable projective Λ-module is superfluous.

(c) The socle of any indecomposable projective Λ-module is indecomposable. (12 points)