☆ Rivisitazione della teoria del Dini
in termini del teorema della funzione inversa

$f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y)=0$ $\quad$ (o più in gen $f(x,y)=c$)

$P_0: (x_0,y_0)$ $\quad f(x_0,y_0)=0$ $\quad$ sia $\frac{\partial f}{\partial y}(x_0,y_0) \neq 0$, loc

$\exists \; y = y(x)$, $y_0 = y(x_0)$, t.c. $f(x, y(x)) = 0$

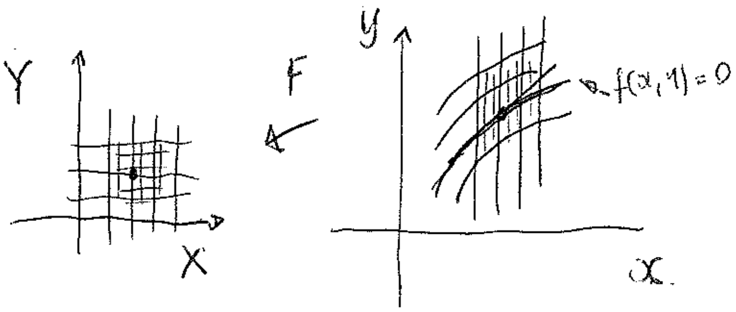
$$F : (x,y) \longmapsto (\overset{X}{x}, \overset{Y}{f(x,y)})$$

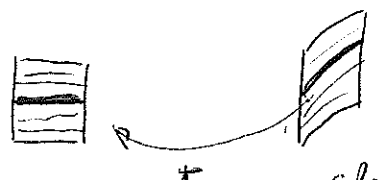
$$\begin{cases} X = x \\ Y = f(x,y) \end{cases} \qquad \begin{aligned} dX &= dx \\ dY &= f_x\, dx + f_y\, dy \end{aligned}$$

$$F_* : \begin{pmatrix} 1 & 0 \\ f_x & f_y \end{pmatrix}$$

$f_y^0 \neq 0 \Rightarrow F_*^0$ isomorfismo

$\Rightarrow F$ è loc. un diffeomorfismo





slice chart

1 - fetta in $\mathbb{R}^2$

$f: \mathbb{R}^3 \to \mathbb{R}$ $\quad f(x,y,z)=0$ $\quad f_z^0 \neq 0$

$f(x_0,y_0,z_0)=0$

$$F : (x,y,z) \longmapsto (\overset{X}{x}, \overset{Y}{y}, \overset{Z}{f(x,y,z)})$$

$$F_* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ f_x & f_y & f_z \end{pmatrix}$$

$f_z^0 \neq 0 \Rightarrow F_*^0$ isomorfismo

$\Rightarrow F$ loc. diffeomorfismo

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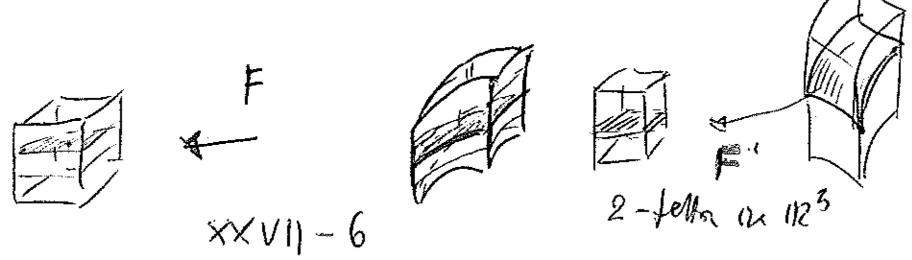


2 - fetta in $\mathbb{R}^3$

$F: \quad \mathbb{R}^3 \to \mathbb{R}^3$

$\begin{cases} f=0 \\ g=0 \end{cases}$

curva in $\mathbb{R}^3$

$(x,y,z) \mapsto (\overset{X}{x}, \overset{Y}{f(x,y,z)}, \overset{Z}{g(x,y,z)})$

$dX = dx$

$dY = f_x\,dx + f_y\,dy + f_z\,dz$

$dZ = g_x\,dx + g_y\,dy + g_z\,dz$

$X = \mathbb{R}^3$

$Y = \mathbb{R}^2$

$f = c_1 \quad (x,y,z) \xmapsto{(f,g)} (c_1,c_2)$

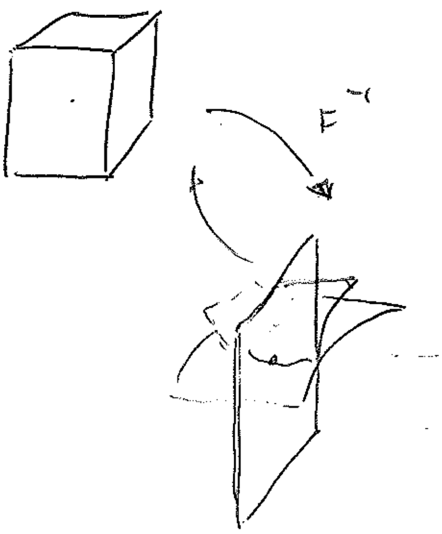
$g = c_2$

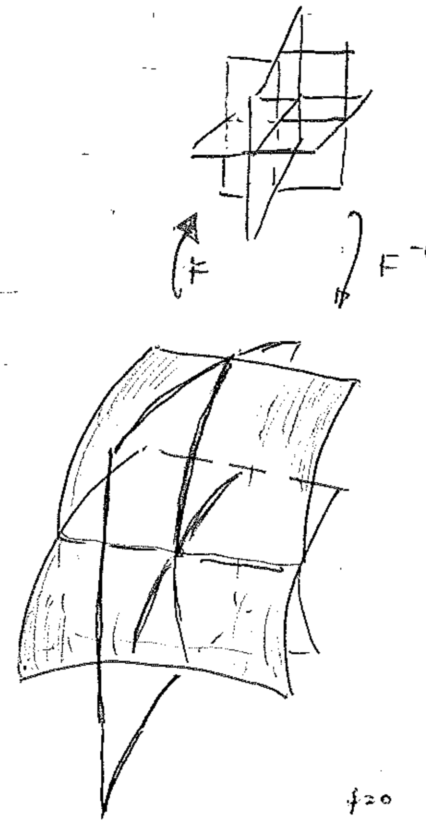
$$F_* = \begin{pmatrix} 1 & 0 & 0 \\ f_x & f_y & f_z \\ g_x & g_y & g_z \end{pmatrix}$$

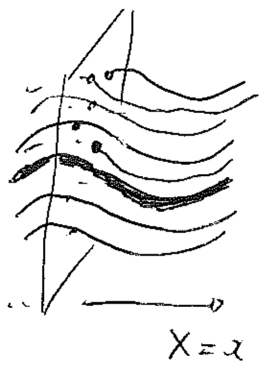
$\frac{\partial(f,g)}{\partial(y,z)}(x_0 y_0 z_0) \neq 0$

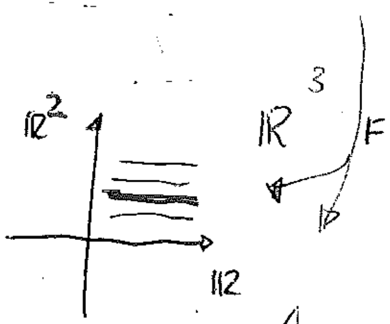
$\Rightarrow F^0_*$ isom

$\Rightarrow F$ diffeom loc

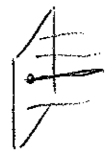


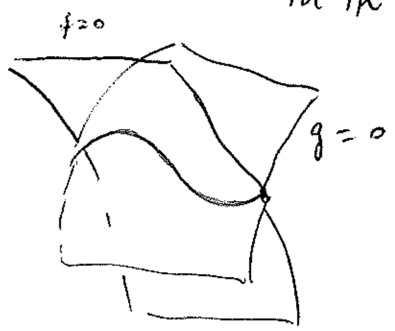






1-fetta in $\mathbb{R}^3$





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