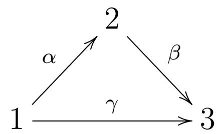
7.5 Exercises - Part 4

(published on November 29, **solutions to be submitted on December 13, 2016**).

Exercise 13. Let K be a field and Q the quiver



(a) Determine all indecomposable projective representations and their radicals.

(b) Determine all indecomposable injective representations and their socles.

(c) Determine the minimal projective resolutions of the simple modules.

(d) Compute the representation $\nu(S_1)$.

(e) Compute the representation $\tau(S_1)$.

Exercise 14. Let Λ be a finite dimensional algebra over a field k, let M, N be finitely generated Λ-modules without projective summands, and let $f \in \mathrm{Hom}_\Lambda(M, N)$. Show

(a) $f \in P(M, N)$ if and only if $\mathrm{Tr}\, f \in P(\mathrm{Tr}\, N, \mathrm{Tr}\, M)$.

(b) f is an isomorphism if and only if so is $\mathrm{Tr}\, f$.

Show further that $\underline{\mathrm{Hom}}_\Lambda(M, N) \to \underline{\mathrm{Hom}}_\Lambda(\mathrm{Tr}\, N, \mathrm{Tr}\, M)$, $\underline{f} \mapsto \underline{\mathrm{Tr}\, f}$ is an isomorphism of k-vector spaces.

Exercise 15. Given a pair of homomorphisms in RMod

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ \downarrow g & & \\ C & & \end{array}$$

consider the cokernel L of the map $A \to B \oplus C$, $a \mapsto (f(a), -g(a))$. Prove that

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ \downarrow g & & \downarrow \tau \\ C & \xrightarrow{\sigma} & L \end{array}$$

is a push-out, where $\sigma : C \to L$, $c \mapsto \overline{(0, c)}$, and $\tau : B \to L$, $b \mapsto \overline{(b, 0)}$.

Exercise 16. Given a pair of homomorphisms in RMod

$$\begin{array}{ccc} & & B \\ & & \downarrow f \\ C & \xrightarrow{g} & A \end{array}$$

construct the pull-back of f and g.