Componenti polari della velocità:

\[ \vec{r}(t) = r(t)\vec{u}_r(t) \]

\[ \vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{dr(t)}{dt} \vec{u}_r + r(t) \frac{d\vec{u}_r(t)}{dt} \]

\[ \Rightarrow \vec{v}(t) = \frac{dr(t)}{dt} \vec{u}_r + r(t) \frac{d\vartheta(t)}{dt} \vec{u}_\vartheta \]

\[ \vec{v}(t) = \left( \frac{dr(t)}{dt}, r(t) \frac{d\vartheta(t)}{dt} \right) = \left( \frac{dx(t)}{dt}, \frac{dy(t)}{dt} \right) \]

“velocità radiale”

“velocità trasversa”

componenti polari

componenti cartesiane
Componenti polari dell’accelerazione

\[
\ddot{a} \equiv \frac{d\ddot{v}(t)}{dt} = \frac{d}{dt} \left( \frac{dr}{dt} \ddot{u}_r + r \frac{d\vartheta}{dt} \ddot{u}_\vartheta \right) = \\
= \frac{d^2r}{dt^2} \dddot{u}_r + \frac{dr}{dt} \frac{d\ddot{u}_r}{dt} + \frac{dr}{dt} \frac{d\vartheta}{dt} \ddot{u}_\vartheta + r \frac{d^2\vartheta}{dt^2} \dddot{u}_\vartheta + r \frac{d\vartheta}{dt} \frac{d\ddot{u}_\vartheta}{dt}
\]

\[
\Rightarrow \quad \ddot{a} = \left( \frac{d^2r}{dt^2} - r \left( \frac{d\vartheta}{dt} \right)^2 \right) \dddot{u}_r + \left( 2 \frac{dr}{dt} \frac{d\vartheta}{dt} + r \frac{d^2\vartheta}{dt^2} \right) \ddot{u}_\vartheta
\]

In un moto circolare (r = costante):

\[
a_r = -r \left( \frac{d\vartheta}{dt} \right)^2 = -r \omega^2 \equiv -a_N
\]

\[
a_\vartheta = r \frac{d^2\vartheta}{dt^2} = r \frac{d\omega}{dt} = r \alpha \equiv a_T
\]

“accelerazione radiale”  “accelerazione trasversa”
Moto circolare uniforme:

velocità con modulo costante:

\[ v(t) = \omega R \mathbf{u}_T (t) \]

coordinata curvilinea
\[ s(t) = R \vartheta (t) \]

"velocità angolare"
\[ \omega = \frac{d\vartheta(t)}{dt} \]

\[ \vartheta(t) = \vartheta_0 + \omega t \]

\[ x(t) = R \cos \vartheta(t) \]
\[ y(t) = R \sin \vartheta(t) \]

\[ v_x(t) = \frac{dx(t)}{dt} = -R \sin \vartheta(t) \frac{d\vartheta}{dt} \equiv -R \omega \sin \vartheta(t) \]
\[ v_y(t) = \frac{dy(t)}{dt} = R \cos \vartheta(t) \frac{d\vartheta}{dt} \equiv R \omega \cos \vartheta(t) \]

\[ \vec{v}(t) = (v_x(t), v_y(t)) = R \omega (-\sin \vartheta(t), \cos \vartheta(t)) \]

\[ \vec{v}(t) = R \omega \mathbf{u}_T (t) = v \mathbf{u}_T (t) \]

\[ a_x(t) = \frac{dv_x(t)}{dt} = -R \omega \cos \vartheta(t) \frac{d\vartheta}{dt} = -R \omega^2 \cos \vartheta(t) \]
\[ a_y(t) = \frac{dv_y(t)}{dt} = -R \omega \sin \vartheta(t) \frac{d\vartheta}{dt} = -R \omega^2 \sin \vartheta(t) \]

\[ \vec{a}(t) = (a_x(t), a_y(t)) = R \omega^2 (-\cos \vartheta(t), -\sin \vartheta(t)) \]

\[ \vec{a}(t) = R \omega^2 \mathbf{u}_N (t) = \frac{v^2}{R} \mathbf{u}_N (t) \]