Box 13-1 RELATIONSHIP BETWEEN SINES, COSINES, AND EXPONENTIALS

It is possible to express periodically varying functions either in terms of sines and cosines or as complex exponentials. The basic relationship between these two representations is

$$e^{ix} = \cos x + i \sin x$$

One easy way to justify this relationship is to expand each of the functions in an infinite series:

$$e^{ix} = 1 + ix - x^2/2! - ix^3/3! + x^4/4! + ix^5/5! - \cdots$$

$$\cos x = 1 - x^2/2! + x^4/4! - x^6/6! + \cdots$$

$$i \sin x = ix - ix^3/3! + ix^5/5! - ix^7/7! + \cdots$$

Because $\cos(-x) = \cos x$, and $\sin(-x) = -\sin x$, it is obvious that

$$e^{-ix} = \cos x - i \sin x$$

Therefore, we can always represent trigonometric functions in terms of complex exponentials as follows:

$$\cos x = (1/2)(e^{ix} + e^{-ix})$$

$$\sin x = (1/2i)(e^{ix} - e^{-ix})$$