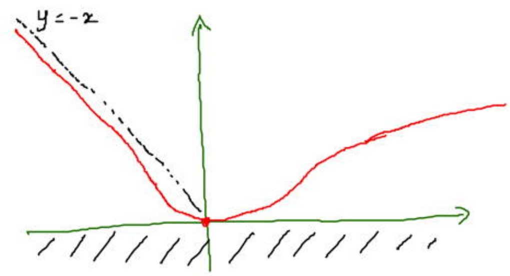
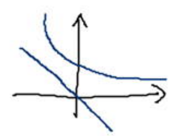
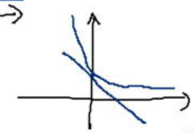
MATEMATICA
Università di Verona
Laurea in Biotecnologie - A.A. 2012/13
Lezione di venerdì 23/11/2012

$f(x) = \log(x+e^{-x})$



$x+e^{-x}>0 \qquad e^{-x}>-x$
sempre!





$f(x)=0 \qquad x+e^{-x}=1 \qquad e^{-x}=1-x \qquad x=0$

$f(x)>0 \qquad x+e^{-x}>1 \qquad e^{-x}>1-x \qquad x\neq 0$

$\lim\limits_{x\to-\infty} f(x) \overset{(F.I.)}{=} +\infty \qquad \lim\limits_{x\to-\infty}(x+e^{-x}) = \lim\limits_{t\to+\infty}(e^{t}-t) = \lim\limits_{t\to+\infty} e^{t}\Big(1-\overset{0}{\frac{t}{e^{t}}}\Big) = +\infty$

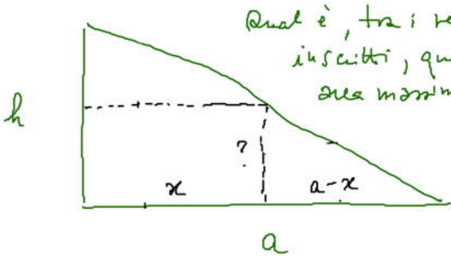
$\lim\limits_{x\to+\infty} f(x) = +\infty \qquad f'(x) = \frac{1}{x+e^{-x}}\cdot(1-e^{-x})$

$f'(x)=0 \quad e^{-x}=1 \quad -x=0 \quad x=0 \qquad f'(x)>0 \quad e^{-x}<1 \quad -x<0 \quad x>0$

		0	
f'	$-$		$+$
f	$\searrow$	min	$\nearrow$

$f(0)=0$ min ass.

Qual è, tra i rettangoli inscritti, quello di area massima?



x = base del rettangolo $\quad 0\le x\le a$

$f(x)$ = area del rettangolo

$a : h = (a-x) : ? \qquad ? = \frac{a-x}{a}h$

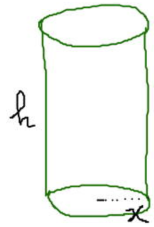
$f(x) = x\cdot\frac{a-x}{a}h = \frac{h}{a}x(a-x) \qquad f'(x) = \frac{h}{a}(a-2x)$

$f'(x)=0 \quad x=a/2 \qquad f'(x)>0 \quad x<a/2$

	0		$a/2$		a
f'		$+$		$-$	
f		$\nearrow$	MAX	$\searrow$	

$x = a/2 \qquad f(a/2) = \frac{ha}{4}$

pentola cilindrica di data superficie interna S.
Quanto deve essere alta la pentola affinché abbia la max capienza possibile?



$f(x) =$ volume $\qquad S = x^2\pi + (2x\pi)h \qquad h = \dfrac{S - x^2\pi}{2\pi x}$

$$f(x) = x^2\pi\, h = x^{\cancel{2}}\cancel{\pi} \cdot \frac{S - x^2\pi}{2\cancel{\pi}\cancel{x}} = \frac{1}{2}x\,(S - x^2\pi)$$

$x^2\pi \leq S \qquad x \leq \sqrt{S/\pi} \qquad 0 \leq x \leq \sqrt{\dfrac{S}{\pi}}$

$f'(x) = \frac{1}{2}(S - 3\pi x^2) \qquad f'(x) = 0 \quad x = \sqrt{\dfrac{S}{3\pi}}$

$f'(x) > 0 \qquad 0 < x < \sqrt{\dfrac{S}{3\pi}}$

	0	$\sqrt{S/3\pi}$	$\sqrt{S/\pi}$
f'	$+$	$-$	
f	↗	↘	
		max	

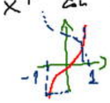
$$h = \frac{S - \frac{S}{3\cancel{\pi}}\cdot\cancel{\pi}}{2\pi\sqrt{S/3\pi}} = \frac{\cancel{2}S/3}{\cancel{2}\sqrt{S\pi/3}} = \sqrt{\frac{S}{3\pi}}$$

$$f\left(\sqrt{S/3\pi}\right) = \frac{1}{2}\sqrt{S/3\pi}\,\left(S - \frac{S}{3\cancel{\pi}}\cancel{\pi}\right) = \frac{1}{\cancel{2}}\sqrt{S/3\pi}\,\frac{\cancel{2}S}{3} = \frac{S}{3}\sqrt{S/3\pi}$$

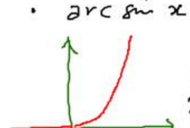
Derivate successive

Più una funzione è regolare, più volte si riesce a derivarla.
In realtà tutte le nostre funzioni elementari sono derivabili infinite volte ($\mathcal{C}^\infty$) nel loro dominio tranne:

- il modulo $|x|$ in $x=0$
- la radice $\sqrt{x}$ in $x=0$ (in generale, x^β con $0<\beta<1$)
- $\arcsin x$ e $\arccos x$ in $x=-1$ opp. $x=1$



$$f(x) = \begin{cases} 0 & \text{se } x \leq 0 \\ x^2 & \text{se } x > 0 \end{cases}$$



$\lim\limits_{x\to 0} \dfrac{f(x)-f(0)}{x-0} = 0 \qquad f'(0) = 0 \qquad f'(x) = \begin{cases} 0 & \text{se } x \leq 0 \\ 2x & \text{se } x > 0 \end{cases}$

Formule di Taylor per funzioni "analitiche" (un po' meglio di $\mathcal{C}^\infty$)
(le funz. elem. sono analitiche)

$f: A \to \mathbb{R} \qquad x_0 \in A \qquad f$ sia analitica (un po' meglio di $\mathcal{C}^\infty$)

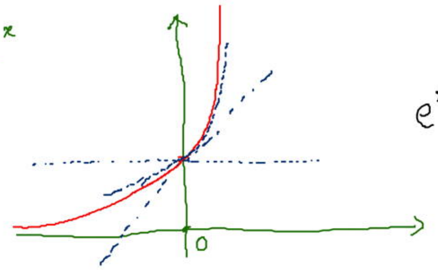
Allora, quando x è vicino a x_0 (cioè, in un intorno di x_0) si può scrivere:

$$\boxed{f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2}(x-x_0)^2 + \cdots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + \cdots}$$

a meno di un errore sempre più piccolo

Ex. $f(x) = e^x$

$x_0 = 0$



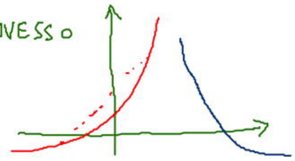
$$f'(x) = f''(x) = f'''(x) = \cdots = e^x$$

$$e^x = e^0 + e^0(x-0) + \frac{e^0}{2}(x-0)^2 + \frac{e^0}{3!}(x-0)^3 + \cdots$$

$$= \underline{1 + x + x^2/2 + x^3/6 + \cdots}$$

Derivata seconda: derivata della derivata = come cresce o decresce la pendenza della funzione

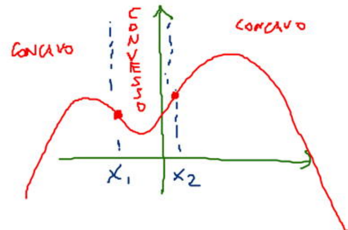
CONVESSO



in questi casi la pendenza sta crescendo

CONCAVO

e qui sta decrescendo



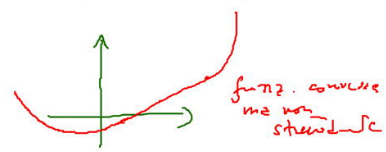
Punti di passaggio tra zone di concavità e convessità: FLESSI

Prop. Sia $f: A \to \mathbb{R}$ derivabile almeno 2 volte (ci gen: C^∞)

(a) Se $c \in A$ è flesso $\Rightarrow f''(c) = 0$

(b) f strett. convessa $\Leftarrow f''(c) > 0$

strett. concava $\Leftarrow f''(c) < 0$



Es. $f(x) = x^3 - x$

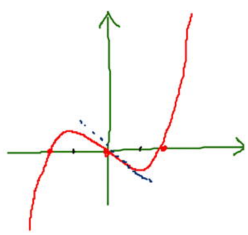
dispari

$f(x) = 0 \quad x = 0$

$x = \pm 1$

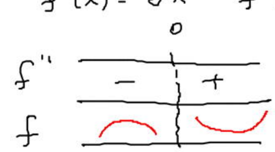
$f(x) > 0 \quad -1 < x < 0$

$x > 1$



$f'(x) = 3x^2 - 1 \quad f'(x) = 0 \quad x = \pm \sqrt{3}/3 \quad f\left(\pm\frac{\sqrt{3}}{3}\right) = \mp\frac{2}{3\sqrt{3}}$

$f''(x) = 6x \quad f''(x) = 0 \quad x = 0 \quad f''(x) > 0 \quad x > 0$

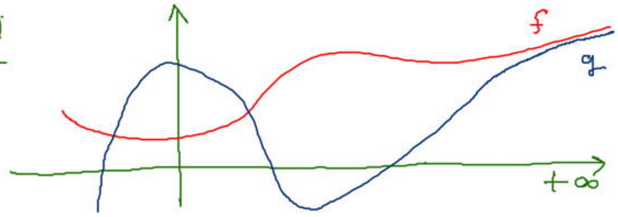


$x = 0$ è flesso.

$f(0) = 0$

$f'(0) = -1$

ASINTOTI

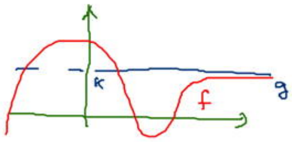


Si dice che g è asintoto per f a $+\infty$ quando

$$\lim_{x\to+\infty}\big(f(x)-g(x)\big)=0$$

Quando è che una funzione $f(x)$ ha come asintoto a $+\infty$ una retta?

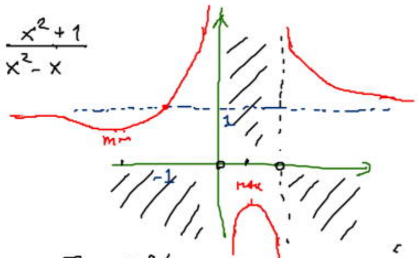
- $g(x)=K$



$$\lim_{x\to+\infty}\big(f(x)-K\big)=0 \qquad \lim_{x\to+\infty} f(x)=K\in\mathbb{R}$$

(asintoto orizzontale $y=K$)

Ex. $f(x)=\dfrac{x^2+1}{x^2-x}$



$x\neq 0,1$ $\quad f(x)=0$ mai $\quad f(x)>0 \quad x<0 \lor x>1$

$\lim_{x\to+\infty} f(x)=1 \qquad \dfrac{x^2+1}{x^2-x}=1 \quad x^2+1=x^2-x \quad x=-1$

$\lim_{x\to-\infty} f(x)=1 \qquad f'(x)=\dfrac{2x(x^2-x)-(2x-1)(x^2+1)}{(x^2-x)^2}$

$$=\frac{\cancel{2x^3}-2x^2-\cancel{2x^3}-2x+x^2+1}{(x^2-x)^2}=-\frac{x^2+2x-1}{(x^2-x)^2}$$

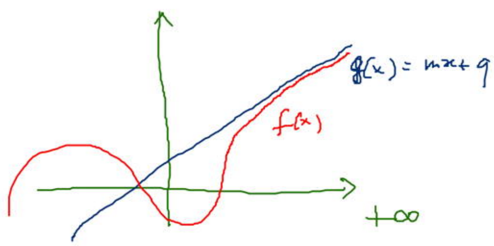
$f'(x)=0 \qquad x=-1\pm\sqrt{2}$ $\quad \nearrow \sim 0{,}4$ $\quad \searrow \sim -2{,}4$

$f'(x)>0 \quad x^2+2x-1<0 \qquad -1-\sqrt{2}<x<-1+\sqrt{2}$

$f(-1-\sqrt{2})=\dots \qquad f(-1+\sqrt{2})=\dots$

		$-1-\sqrt2$		0		$-1+\sqrt2$		1	
f'	$-$		$+$		$+$		$-$		$-$
f	$\searrow$	min	$\nearrow$		$\nearrow$	max	$\searrow$		$\searrow$

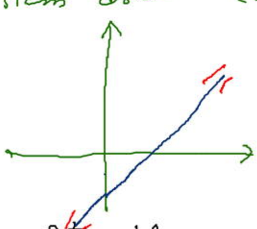
- $g(x)=mx+q$



$f(x)$ ha $g(x)=mx+q$ come asintoto obliquo a $+\infty$ $\iff$ $\begin{cases}\lim_{x\to+\infty}\dfrac{f(x)}{x}=m\in\mathbb{R}\\ \lim_{x\to+\infty}\big(f(x)-mx\big)=q\in\mathbb{R}\end{cases}$

A priori, quello che accade a $+\infty$ non è detto accada anche a $-\infty$
però se $f(x)=\dfrac{P(x)}{Q(x)}$ con P, Q polinomi (funz. razionale)
allora f ha uno stesso asintoto (se c'è) da entrambe le parti

Ex $\quad f(x)=\dfrac{x^2-1}{x+2}$



$\lim_{x\to+\infty}\dfrac{f(x)}{x}=\lim_{x\to+\infty}\dfrac{x^2-1}{x(x+2)}=1 \quad (m)$

$\lim_{x\to+\infty}\big(f(x)-1\cdot x\big)=\lim_{x\to+\infty}\left(\dfrac{x^2-1}{x+2}-x\right)=$

$=\lim_{x\to+\infty}\dfrac{\cancel{x^2}-1-\cancel{x^2}-2x}{x+2}=-2 \quad (q)$

$y=x-2$ asintoto obliquo a $+\infty$ (ma anche a $-\infty$!)

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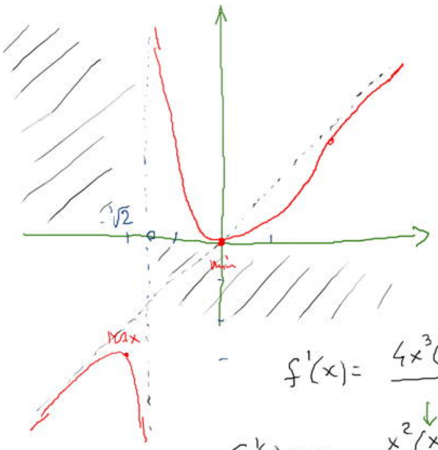
$$f(x) = \frac{x^5}{x^3+2}$$



Dominio: $x^3+2 \neq 0 \quad x \neq -\sqrt[3]{2}$

Parità? $f(-x) = \frac{(-x)^5}{(-x)^3+2} = \frac{x^5}{-x^3+2} \neq f(x), \neq -f(x)$ nessuna

Periodo: nessuno

Zeri: $x = 0$ Limiti interessanti: $\lim_{x \to \mp\infty} f(x) = \pm\infty \quad \lim_{x \to -\sqrt[3]{2}^{\mp}} f(x) = \mp\infty$

Segno: $f(x) > 0 \quad x^3+2 > 0 \quad x > -\sqrt[3]{2}$

$x = -\sqrt[3]{2}$ asintoto vert. bilatero

Asintoto obliquo? $\lim_{x \to +\infty} \frac{f(x)}{x} = 1 = m \quad \lim_{x \to +\infty} (f(x) - 1x) = \lim_{x \to +\infty} \frac{-2x}{x^3+2} \overset{\shortparallel}{0} = q$

$y = x$

$f(x) = x \quad \frac{x^5}{x^3+2} = x \quad \cancel{x^5} = \cancel{x^4} + 2x \quad x = 0$

$$f'(x) = \frac{5x^4(x^3+2) - 3x^2 \cdot x^5}{(x^3+2)^2} = \frac{x^3(x^3+8)}{(x^3+2)^2} \qquad f'(x) = 0 \quad \begin{cases} x = 0 \\ x = -2 \end{cases}$$

$$f'(x) > 0 \quad \frac{x^2(x)(x+2)(x^2-2x+4)}{(x^3+2)^2} > 0 \quad x < -2 \ \vee \ x > 0$$

	-2		$-\sqrt[3]{2}$		0	
f'	+		−	−		+
f	↗		↘	↘		↗
		MAX			min	

$f(0) = 0 \quad f(-2) = -8/3$

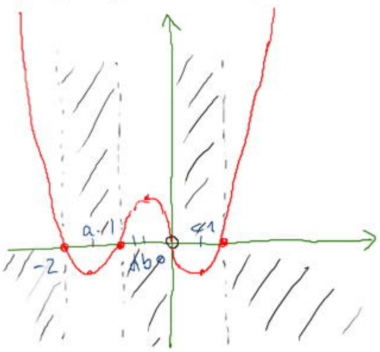
$$f''(x) = \frac{[3x^2(x^3+8) + 3x^2 \cdot x^3](x^3+2)^{\cancel{2}} - x^3(x^3+8) \cdot 2 \cdot 3x^2 \cancel{(x^3+2)}}{(x^3+2)^{\cancel{4}3}} = \frac{x^2}{(x^3+2)^3}\left(3x^3+24+3x^3 - 6x^3(x^3+8)\right)$$

$-6x^6 - 42x^3 + 24$

$-6(x^6 + 7x^3 - 4)$

$x^3 = \frac{-7 \pm \sqrt{49+16}}{2}$

$$f(x) = (x^2+2x)\log|x|$$



Dominio: $x \neq 0$ No parità, periodo.

$f(x) = 0 \quad \begin{cases} x^2+2x = 0 & x = -2, \ \cancel{x = 0} \\ \log|x| = 0 & |x| = 1 \quad x = \pm 1 \end{cases}$

Segno $f(x) > 0$ ① $x^2+2x > 0 \quad x < -2 \ \vee \ x > 0$

② $\log|x| > 0 \quad |x| > 1 \quad x < -1 \ \vee \ x > 1$

	-2		-1		0		1	
①	+		−		−		+	+
②	+		+		−		−	+
f	+		−		+		−	+

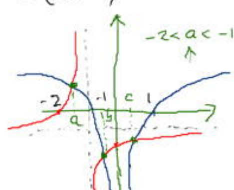
Limiti int. $\lim_{x \to \mp\infty} f(x) = +\infty \quad \lim_{x \to 0^{\mp}} f(x) = \lim_{x \to 0^{\mp}} (x+2) \underbrace{x \log|x|}_{0} = 0^{\pm}$

Cercare gli asintoti è inutile!

$$f'(x) = (2x+2)\log|x| + (x^2+2x) \cdot \frac{1}{\cancel{|x|}} \cdot \frac{\cancel{|x|}}{x} = 2(x+1)\log|x| + (x+2)$$

$f'(x) = 0 \qquad 2(x+1)\log|x| = -(x+2) \qquad$ se $x+1 = 0$ cosa succede? $0 = 1$ NO

$$\log|x| = -\frac{x+2}{2(x+1)}$$



$-2 < a < -1 < b < 0 < c < 1$

	a	-1	b	0	c	
f'	−	+	+	−	−	+
f	↘	↗	↗	↘	↘	↗
	min		MAX		min	

$\lim_{x \to 0} f'(x) = -\infty$

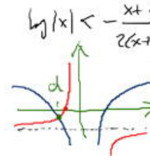
$f'(x) > 0 \qquad 2(x+1)\log|x| > -(x+2)$

se $x+1 > 0$ (cioè $x > -1$) $\quad \log|x| > -\frac{x+2}{2(x+1)} \quad -1 < x < b, \ x > c$

se $x+1 = 0$ (cioè $x = -1$) $\quad 0 > -1$ vero $\quad x = -1$

se $x+1 < 0$ (cioè $x < -1$) $\quad \log|x| < -\frac{x+2}{2(x+1)} \quad a < x < -1$

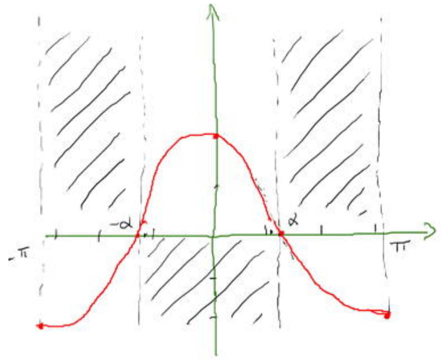
$$f''(x) = 2\log|x| + 2(x+1) \cdot \frac{1}{x} + 1 = 2\log|x| + \frac{3x+2}{x} \qquad f''(x) \gtreqless 0 \quad \log|x| \gtreqless -\frac{3x+2}{2x}$$



$-1 < d < 0$

$f(x) = 2\cos x - \sin^2 x$

Studieremo in $[-\pi, \pi]$



Dominio: $\mathbb{R}$ Parità? $f(-x) = 2\cos(-x) - \sin^2(-x) =$
$= 2\cos x - (\sin(-x))^2 = 2\cos x - (-\sin x)^2$
$= 2\cos x - \sin^2 x = f(x)$ PARI

Periodo: 2π

$f(\pi) = f(-\pi) = -2$ $\quad f(0) = 2$

$f(x) = 0$ $\quad 2\cos x - \sin^2 x = 0$ $\quad 2\cos x - 1 + \cos^2 x = 0$ $\quad \cos^2 x + 2\cos x - 1 = 0$

$t = \cos x$ $\quad t^2 + 2t - 1 = 0$ $\quad t = -1 \pm \sqrt{2}$ $\quad \cos x = \sqrt{2} - 1$ $\quad \alpha = \arccos(\sqrt{2}-1)$

$f(x) > 0$ $\quad \cos^2 x + 2\cos x - 1 > 0$ $\quad t^2 + 2t - 1 > 0$ $\quad \cancel{t < -\sqrt{2}-1}$ $\quad t > \sqrt{2} - 1$ $\quad -\alpha = -\arccos(\sqrt{2}-1)$

$\cos x > \sqrt{2} - 1$ $\quad -\alpha < x < \alpha$

$f'(x) = -2\sin x - 2\sin x \cos x = -2\sin x\,(1 + \cos x)$

$f'(x) = 0$ $\begin{cases} \sin x = 0 & x = -\pi, 0, \pi \\ 1 + \cos x = 0 & x = -\pi, \pi \end{cases}$ $\quad f'(x) > 0$ $\quad \sin x < 0$ $\quad -\pi < x < 0$

$-\pi$		0		π
f'	$+$		$-$	
f	$\nearrow$		$\searrow$	

min MAX min

$f''(x) = -2\cos x - 2\cos 2x = -2\cos x - 2(2\cos^2 x - 1) = -2(2\cos^2 x + \cos x - 1)$

$f''(x) = 0$ $\quad 2\cos^2 x + \cos x - 1 = 0$ $\quad 2t^2 + t - 1 = 0$ $\quad (t = -1) \vee (t = 1/2)$ $\quad \cos x = -1$ $\quad x = -\pi, \pi$

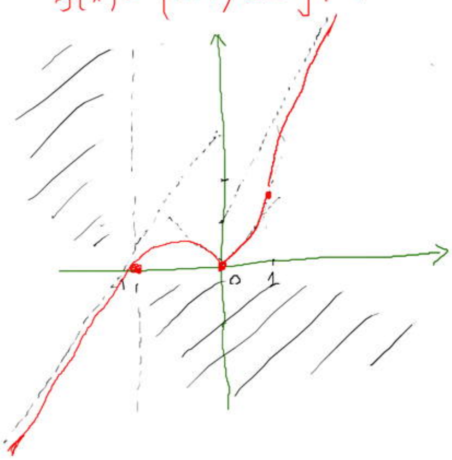
$\cos x = 1/2$ $\quad x = -\pi/3, \pi/3$

$f''(x) > 0$ $\quad$ " $\quad$ " < 0 $\quad$ " $\quad$ " < 0 $\quad -1 < t < 1/2$ $\quad -\pi < x < -\pi/3$ $\quad \pi/3 < x < \pi$

$-\pi$		$-\pi/3$		$\pi/3$		π
f''	$+$		$-$		$+$	
f	$\cup$		$\cap$		$\cup$	

$f(\pm\pi/3) = 2 \cdot 1/2 - (\sqrt{3}/2)^2 = 1/4$ $\quad f'(\pi/3) = -2 \cdot \sqrt{3}/2\,(1 + 1/2) = -\sqrt{3} \cdot 3/2 = -\frac{3\sqrt{3}}{2} \sim -2{,}6$

$f(x) = (x+1)\,\mathrm{arctg}\,|x|$



Dominio $\mathbb{R}$. Parità, periodo: no.

Zeri $f(x) = 0$ $\begin{cases} x + 1 = 0 & x = -1 \\ \mathrm{arctg}|x| = 0 & |x| = 0 \quad x = 0 \end{cases}$

Segno $f(x) > 0$ $\quad x + 1 > 0$ $\quad x > -1$
$\mathrm{arctg}\,|x| > 0$ $\quad |x| > 0$ sempre (purché $x \neq 0$)

Limiti: $\pm\infty$ $\quad \lim_{x \to \pm\infty} f(x) = \pm\infty$

Asintoti $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \underbrace{\frac{x+1}{x}}\,\underbrace{\mathrm{arctg}(x)}_{\pi/2} = \pi/2$

$\lim_{x\to+\infty} \left(f(x) - \pi/2\, x\right) = \lim_{x\to+\infty} \left[(x+1)\,\mathrm{arctg}\,x - \pi/2\, x\right]$

$= \lim_{x\to+\infty} \left(x\,\mathrm{arctg}\,x + \underbrace{\mathrm{arctg}\,x}_{\pi/2} - \pi/2\, x \right)$

$= \lim_{x\to+\infty} \left(\mathrm{arctg}\,x + x\,(\mathrm{arctg}\,x - \pi/2) \right)$

$= \pi/2 - 1$

$\lim_{x\to+\infty} x\,(\mathrm{arctg}\,x - \pi/2) = \lim_{x\to+\infty} \frac{\mathrm{arctg}\,x - \pi/2}{1/x} = \lim_{x\to+\infty} \frac{\frac{1}{1+x^2}}{-1/x^2} = -1$

$y = \pi/2\, x + \pi/2 - 1$ $\quad$ asintoto obliquo a $+\infty$

Asintoto a $-\infty$: $\lim_{x\to-\infty} \frac{f(x)}{x} = \pi/2$

$\lim_{x\to-\infty} \left((x+1)\,\mathrm{arctg}\,|x| - \pi/2\, x\right) =$

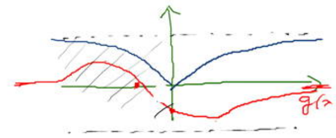
$\lim_{x\to-\infty} \left(\mathrm{arctg}\,|x| + x\,(\mathrm{arctg}\,|x| - \pi/2) \right)$

$= \pi/2 + 1$ $\quad y = \pi/2\, x + \pi/2 + 1$ $\quad$ asintoto obliquo a $-\infty$

$f'(x) = \mathrm{arctg}\,|x| + (x+1) \cdot \frac{1}{1+x^2}\,(\mathrm{sgn}\,x)$

$= \begin{cases} \mathrm{arctg}\,x + \frac{x+1}{x^2+1} & \text{se } x > 0 \\ \mathrm{arctg}\,|x| - \frac{x+1}{x^2+1} & \text{se } x < 0 \end{cases}$

$f'(x)=0$ caso $x>0$: $\quad \operatorname{arctg}|x| = -\underbrace{\dfrac{x+1}{x^2+1}}_{g(x)}$ mai

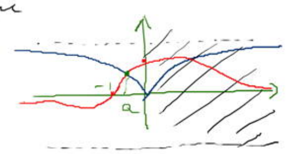


$g'(x) = -\dfrac{1(x^2+1)-2x(x+1)}{(x^2+1)^2} = -\dfrac{x^2+1-2x^2-2x}{(x^2+1)^2} = \dfrac{x^2+2x-1}{(x^2+1)^2}$

$g'(x)=0 \quad x=-1\pm\sqrt{2}$

$f'(x)>0 \quad \operatorname{arctg}|x| > -\dfrac{x+1}{x^2+1}$ sempre

caso $x<0$ $\qquad f'(x)=0 \quad \operatorname{arctg}|x| = \underbrace{\dfrac{x+1}{x^2+1}}_{h(x)}$



$h(-\sqrt{2}-1) = \dfrac{-\sqrt{2}-1+1}{2+1+2\sqrt{2}+1} = \dfrac{-\sqrt{2}}{4+2\sqrt{2}}$

$-1<a<0$

$f'(x)>0 \quad \operatorname{arctg}|x| > \dfrac{x+1}{x^2+1} \qquad x<a$

		a		0	
f'	$+$		$-$		$+$
f	$\nearrow$	MAX	$\searrow$	min	$\nearrow$

$\lim_{x\to 0} f'(x) = \lim_{x\to 0^{\mp}} \Big(\underbrace{\operatorname{arctg}|x|}_{\to 0} + \underbrace{\dfrac{x+1}{x^2+1}}_{\to 1} \underbrace{(\operatorname{sign} x)}_{\mp 1} \Big) = \mp 1$

per $x>0$ $\qquad f'(x) = \operatorname{arctg} x + \dfrac{x+1}{x^2+1} \qquad f''(x) = \dfrac{1}{x^2+1} + \dfrac{1(x^2+1)-2x(x+1)}{(x^2+1)^2} = \dfrac{\cancel{x^2}+1+\cancel{x^2}+1-\cancel{2x^2}-2x}{(x^2+1)^2}$

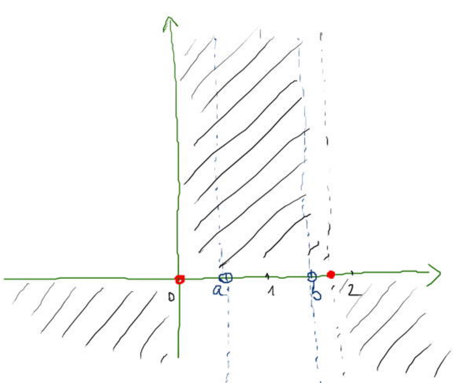
$f''(x) = \dfrac{2(1-x)}{(x^2+1)^2} \qquad f''(x)=0 \quad x=1 \qquad f''(x)>0 \quad 0<x<1$

	0		1	
f''		$+$		$-$
f		$\cup$		$\cap$

$f(1) = 2\operatorname{arctg} 1 = \pi/2$

$f'(1) = \pi/4 + 1 \sim 1{,}75$

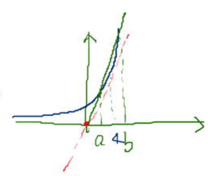
$f(x) = \log|e^x - 3x|$



Dominio: $e^x \neq 3x$

$x\neq a, \; x\neq b$

$0<a<1<b<2$



$y - e^c = e^c(x-c)$

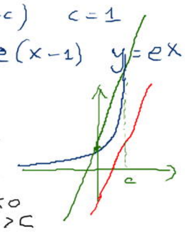
PASSAGGIO PER $(0,0)$

$-e^c = e^c(-c) \qquad c=1$

$y - e = e(x-1) \qquad y = ex$

$f(x)=0 \qquad |e^x - 3x| = 1 \quad \begin{cases} e^x - 3x = 1 & e^x = 3x+1 \\ e^x - 3x = -1 & e^x = 3x-1 \end{cases}$

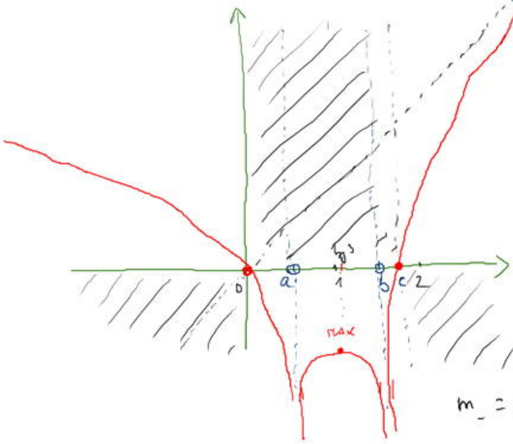
$1<b<c<2$



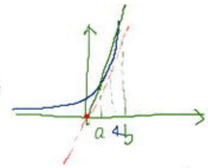
$f(x)>0 \qquad |e^x-3x|>1 \quad \begin{cases} e^x-3x>1 & e^x>3x+1 \quad x<0, \; x>c \\ e^x-3x<-1 & e^x<3x-1 \text{ mai} \end{cases}$

MATEMATICA
Università di Verona
Laurea in Biotecnologie - A.A. 2012/13
Lezione di venerdì 30/11/2012

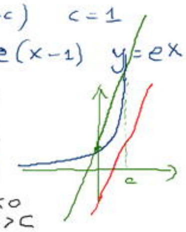
$f(x) = \log|e^x - 3x|$



Dominio: $e^x \neq 3x$
$x \neq a$, $x \neq b$
$0 < a < 1 < b < 2$



$y - e^c = e^c(x - c)$
PASSAGGIO PER $(0,0)$
$-e^c = e^c(-c)$ $\quad c = 1$
$y - e = e(x-1)$ $\quad y = ex$



$f(x) = 0 \qquad |e^x - 3x| = 1 \; \begin{cases} e^x - 3x = 1 & e^x = 3x+1 \\ e^x - 3x = -1 & e^x = 3x - 1 \end{cases}$

$1 < b < c < 2$

$f(x) > 0 \qquad |e^x - 3x| > 1 \; \begin{cases} e^x - 3x > 1 & e^x > 3x+1 \quad x<0,\ x>c \\ e^x - 3x < -1 & e^x < 3x-1 \quad \text{mai} \end{cases}$

limiti interessanti: $-\infty$, $a^\mp$, $b^\mp$, $+\infty$

$\lim_{x\to-\infty} f(x) = +\infty \qquad \lim_{x\to+\infty} f(x) = +\infty \qquad \lim_{\substack{x\to a\\ x\to b}} f(x) = -\infty$

$m_- = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\log|e^x - 3x|}{x} \overset{H}{=} \lim_{x\to-\infty} \frac{\frac{1}{e^x-3x}(e^x-3)}{1} = \lim_{x\to-\infty} \frac{e^x-3}{e^x-3x} = 0^-$

$q_- = \lim_{x\to-\infty} (f(x) - m_- x) = \lim_{x\to-\infty} f(x) = +\infty$ non c'è asintoto obliquo a $-\infty$

$m_+ = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\log(e^x-3x)}{x} \overset{H}{=} \lim_{x\to+\infty} \frac{e^x-3}{e^x-3x} \overset{H}{=} \lim_{x\to+\infty} \frac{e^x}{e^x-3} = 1$

$q_+ = \lim_{x\to+\infty} (f(x) - 1x) = \lim_{x\to+\infty} [\log(e^x - 3x) - \underbrace{x}_{\log e^x}] = \lim_{x\to+\infty} \log\Big(\underbrace{\frac{e^x-3x}{e^x}}_{\to 1}\Big) = 0$

$\Rightarrow$ $y = x$ è asintoto obliquo a $+\infty$

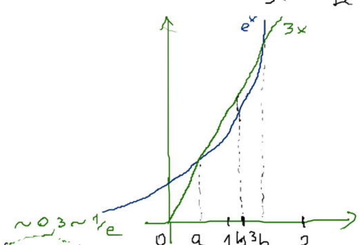
Intersezioni? $\quad f(x) = x \quad \log|e^x - 3x| = x \quad |e^x - 3x| = e^x \; \begin{cases} \cancel{e^x} - 3x = \cancel{e^x} & x = 0 \\ e^x - 3x = -e^x & e^x = \frac{3}{2}x \end{cases}$

mai!

$f'(x) = \frac{e^x - 3}{e^x - 3x} \qquad f'(x) = 0$ se $e^x = 3$ cioè $x = \log 3$; $f(x) > 0$

$N > 0$ se $x > \log 3 \sim 1,1$
$D > 0$ se $x < a$ opp. $x > b$

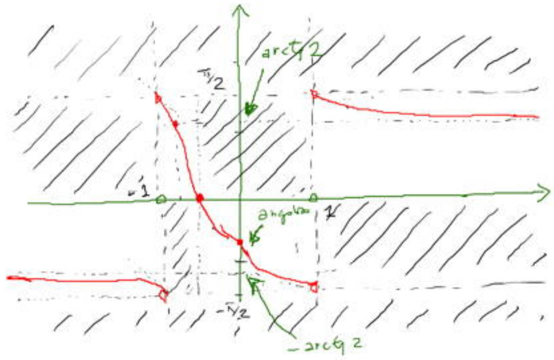
		a		$\log 3$		b	
N	$-$		$-$		$+$		$+$
D	$+$		$-$		$-$		$+$
f'	$-$		$+$		$-$		$+$
f	↘		↗	MAX	↘		↗



$e^{\log 3} = 3$
$3\log 3 \sim 3,3$

$f(\log 3) = \log|e^{\log 3} - 3\log 3| = \log|3 - 3\log 3| = \log(3\log 3 - 3) \sim -1$

$$\boxed{f(x) = \operatorname{arctg}\left(\frac{2x+1}{|x|-1}\right)}$$



Dominio: $|x| \neq 1$ cioè $x \neq \pm 1$, no parità, no periodo.

Visto che sto facendo l'arcotangente di qualcosa, di certo avrò $-\pi/2 < f(x) < \pi/2$, cioè f è limitata tra $-\pi/2$ e $\pi/2$

Zeri: $f(x) = 0 \iff \frac{2x+1}{|x|-1} = 0 \iff 2x+1 = 0 \iff x = -\frac{1}{2}$

Segno: $f(x) > 0 \iff \frac{2x+1}{|x|-1} > 0$ $\quad N > 0 \quad x > -\frac{1}{2}$; $\quad D > 0 \quad |x| > 1 \Rightarrow x < -1 \vee x > 1$

	-1		-1/2		1	
N	−		−	+		+
D	+		−	−		+
f	−		+	−		+

$f(0) = \operatorname{arctg}(-1) = -\pi/4 \sim 0{,}75$

Limiti interessanti: $-\infty, \; -1^{\mp}, \; 1^{\mp}, \; +\infty$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \operatorname{arctg}\left(\frac{2x+1}{-x-1}\right) = \lim_{x\to-\infty} \operatorname{arctg}\left(\frac{\cancel{x}(2+1/x)}{\cancel{x}(-1-1/x)}\right) = \operatorname{arctg}(-2) = -\operatorname{arctg} 2 \sim -1{,}2$$

$$\lim_{x\to+\infty} f(x) = \operatorname{arctg} 2$$

$$\lim_{x\to-1^{\mp}} f(x) = \mp \pi/2 \qquad \lim_{x\to 1^{\mp}} f(x) = \mp\pi/2$$

Intersezioni con gli asintoti orizzontali? $f(x) = \operatorname{arctg} 2$:

$f(x) = \operatorname{arctg} 2 \quad \frac{2x+1}{|x|-1} = 2 \quad 2x+1 = 2|x|-2$ $\begin{cases} (x>0) & 2x+1 = 2x-2 \quad 1 = -2 \quad NO \\ (x<0) & 2x+1 = -2x-2 \quad 4x = -3 \quad x = -3/4 \end{cases}$

$f(x) = -\operatorname{arctg} 2 \quad \frac{2x+1}{|x|-1} = -2 \quad 2x+1 = -2|x|+2$ $\begin{cases} (x>0) & 2x+1 = -2x+2 \quad x = 3/4 \\ (x<0) & 2x+1 = 2x+2 \quad 1 = 2 \quad NO \end{cases}$

$$f'(x) = \frac{1}{1+\left(\frac{2x+1}{|x|-1}\right)^2} \cdot \frac{2(|x|-1) - \sigma(2x+1)}{(|x|-1)^2} \quad [\text{ove } \sigma = \operatorname{sign} x \text{ dunque } \sigma x = |x|] = -\frac{2+\sigma}{5x^2+(4-2\sigma)x+2} \begin{cases} -\frac{1}{5x^2+6x+2} & (\text{se } x<0) \\ -3/5x^2+2x+2 & (\text{se } x>0) \end{cases}$$

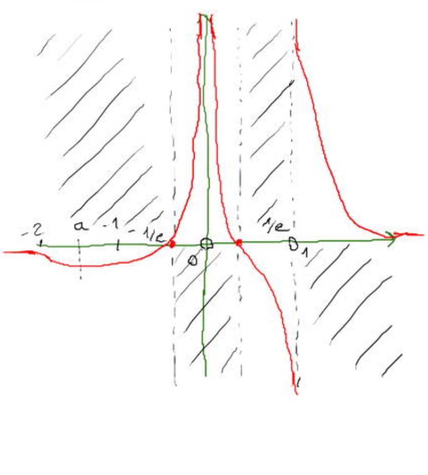
$\lim_{x\to-1} f'(x) = -1$

$\lim_{x\to1} f'(x) = -1/3$

$\left.\begin{array}{l}\lim_{x\to0^-} f'(x) = -1/2 \\ \lim_{x\to0^+} f'(x) = -3/2\end{array}\right\}$ pto angoloso

$\underline{f' \text{ sempre} < 0}$ $\Rightarrow f$ strett. decr.

$$\boxed{f(x) = \frac{1+\lg|x|}{x-1}}$$



Dominio: $x \neq 0, \; x \neq 1$ $\quad$ No simmetrie, no periodo.

Zeri: $f(x) = 0$ per $1 + \lg|x| = 0 \quad \lg|x| = -1 \quad |x| = 1/e \quad x = \pm\frac{1}{e}$

Segno: $f(x) > 0 \quad N > 0 \quad \lg|x| > -1 \quad |x| > 1/e : x < -\frac{1}{e} \vee x > 1/e$

$D > 0 \quad x > 1$

		-1/e		0		1/e		1	
N	+		−		−		+		+
D	−		−		−		−		+
f	−		+		+		−		+

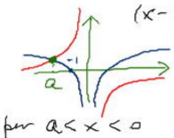
Limiti: $-\infty, \; 0^{\mp}, \; +\infty, \; 1^{\mp}$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{\lg|x|\left(1+\frac{1}{\lg|x|}\right)}{x\,\left(1-\frac{1}{x}\right)} = 0^- \qquad \lim_{x\to+\infty} f(x) = 0^+$$

$$\lim_{x\to0} f(x) = +\infty \qquad \lim_{x\to1^{\mp}} f(x) = \mp\infty$$

$$f'(x) = \frac{\frac{1}{x}(x-1) - 1(1+\lg|x|)}{(x-1)^2} = \frac{\cancel{1} - 1/x - \cancel{1} - \lg|x|}{(x-1)^2} = -\frac{\lg|x| + 1/x}{(x-1)^2}$$

$f'(x) = 0 \iff \lg|x| = -1/x$



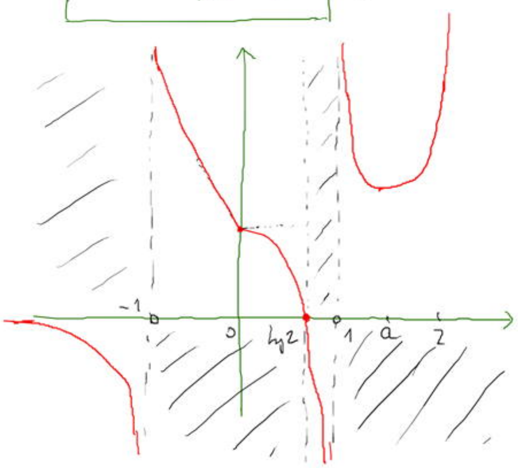
$-2 < a < -1$

$f'(x) > 0 \iff \lg|x| < -1/x$ per $a < x < 0$

		a		0		1	
f'	−		+		−		−
f	↘		↗		↘		↘
		min					

$$\boxed{f(x)=\frac{e^x-2}{|x|-1}}$$

$f(0)=1$



Dominio: $|x|\neq 1$ cioè $x\neq\pm 1$ No simmetrie, periodi

Zeri: $e^x=2$ $x=\lg 2\sim 0{,}7$

Segno: $N>0$ $x>\lg 2$

$D>0$ $x<-1 \vee x>1$

	-1		$\lg 2$		1	
N	$-$		$-$	$+$		$+$
D	$+$		$-$	$-$		$+$
f	$-$		$+$	$-$		$+$

Limiti: $\lim_{x\to-\infty} f(x)=0^-$ $\lim_{x\to+\infty} f(x)=+\infty$ $\lim_{x\to -1^{\mp},\,1^{\mp}} f(x)=\mp\infty$

Ev. asintoto obliquo a $+\infty$ $\lim_{x\to+\infty}\frac{f(x)}{x}=\lim_{x\to+\infty}\frac{e^x-2}{x(x-1)}=+\infty$ No

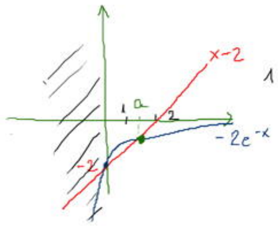
$$f'(x)=\frac{e^x(|x|-1)-\sigma(e^x-2)}{(|x|-1)^2}$$

(ove $\sigma=\operatorname{sgn} x=\begin{cases}1 & \text{se } x>0\\ -1 & \text{se } x<0\end{cases}$ $\sigma x=|x|$)

$$=\frac{\sigma x e^x-e^x-\sigma e^x+2\sigma}{(|x|-1)^2}=\begin{cases}(x>0) & \frac{(x-2)e^x+2}{(x-1)^2} \quad \lim_{x\to0^+}=0\\ (x<0) & \frac{-xe^x-2}{(x+1)^2} \quad \lim_{x\to0^-}=-2\end{cases}$$

$$f'(x)=0 \quad \begin{cases}(x>0) & (x-2)e^x+2=0 \quad x-2=-2e^{-x}\\ (x<0) & -xe^x-2=0 \quad x=-2e^{-x}\end{cases}$$

se $x>0$:



$1<a<2$

$f'(x)>0$

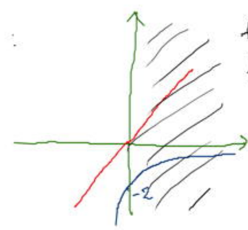
$(x-2)e^x+2>0$

$(x+2)e^x>-2$

$x+2>-2e^{-x}$

$x>a$

se $x<0$:



$f'(x)=0$ mai

$f'(x)>0$ $-xe^x-2>0$

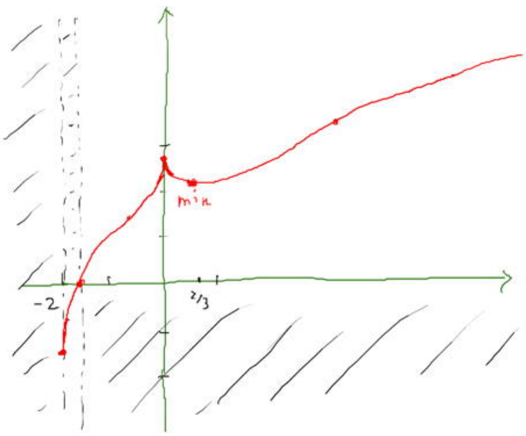
$xe^x+2<0$

$xe^x<-2$

$x<-2e^{-x}$ mai

dunque per $x<0$ $f(x)$ è strett. decrescente

$$\boxed{f(x)=2\sqrt{x+2}-\sqrt{|x|}}$$

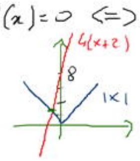


Dominio: $x+2\geq 0$ $x\geq -2$ $f(-2)=-\sqrt2$ $f(0)=2\sqrt2$

No simmetrie, no periodi.

f è continua; f derivabile ovunque tranne che in -2^+ e in 0
(punti in cui mi attendo pendenza infinita)

Zeri. $f(x)=0 \iff 2\sqrt{x+2}=\sqrt{|x|} \iff 4(x+2)=|x|$



Per $x<0$: $4(x+2)=-x$ $5x=-8$ $x=-8/5$

$f(x)>0 \iff 2\sqrt{x+2}>\sqrt{|x|} \iff 4(x+2)>|x| \iff x>-8/5$

Limiti: $\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\frac{4(x+2)-x}{2\sqrt{x+2}+\sqrt{x}}=\lim_{x\to+\infty}\frac{3x+8}{2\sqrt{x+2}+\sqrt x}$

$$\overset{H}{=}\lim_{x\to+\infty}\frac{3}{2\cdot\frac{1}{2\sqrt{x+2}}+\frac{1}{2\sqrt x}}=+\infty$$

0

Asintoto obliquo? $\lim_{x\to+\infty}\frac{f(x)}{x}=\lim_{x\to+\infty}\frac{2\sqrt{x+2}-\sqrt x}{x}\overset{H}{=}0$ no.

$f'(x)=\cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{x+2}}-\frac{1}{2\sqrt{|x|}}\cdot\sigma=\frac{2\sqrt{|x|}-\sigma\sqrt{x+2}}{2\sqrt{|x|(x+2)}}$ Notiamo che per $x<0$ (cioè $\sigma=-1$) si ha che $f'(x)>0$ dunque per $x<0$ la funzione è strett. crescente!

Se ora $x>0$ $f'(x)=\frac{2\sqrt x-\sqrt{x+2}}{2\sqrt{x(x+2)}}$ $f'(x)=0 \iff 2\sqrt x=\sqrt{x+2} \iff 4x=x+2$ $x=2/3$

$f'(x)>0 \iff \cdots > \cdots \iff x>2/3$ $x=2/3$ è pto di minimo

$f(2/3)=\sqrt6$

Si noti che $\lim_{x\to-2^+}f'(x)=-\infty$ $\lim_{x\to0^\pm}f'(x)=\pm\infty$ Si può fare la f'' ...