

Image interpolation

A reinterpretation of low-pass filtering

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Image Interpolation

- Introduction
 - What is image interpolation? (D-A conversion)
 - Why do we need it?
- Interpolation Techniques
 - 1D zero-order, first-order, third-order
 - 2D = two sequential 1D (divide-and-conquer)
 - Directional(Adaptive) interpolation*
- Interpolation Applications
 - Digital zooming (resolution enhancement)
 - Image inpainting (error concealment)
 - Geometric transformations (where your imagination can fly)

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Introduction

- What is image interpolation?
 - An image $f(x,y)$ tells us the intensity values at the integral lattice locations, i.e., when x and y are both integers
 - Image interpolation refers to the “guess” of intensity values at missing locations, i.e., x and y can be arbitrary
 - Note that it is just a guess (Note that all sensors have finite sampling distance)

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A Sentimental Comment

- Haven't we just learned from discrete sampling (A-D conversion)?
- Yes, image interpolation is about D-A conversion
- Recall the gap between biological vision and artificial vision systems
 - Digital: camera + computer
 - Analog: retina + brain

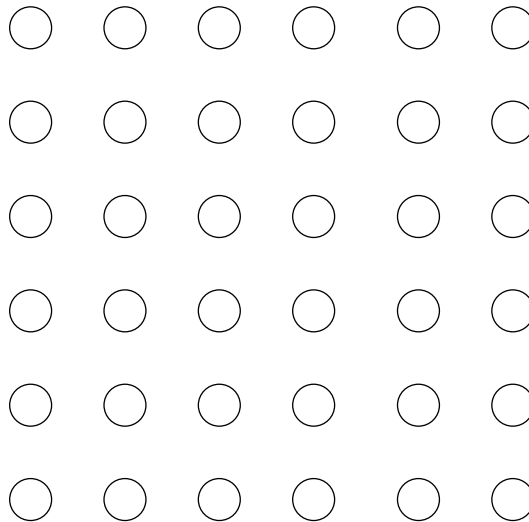
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Engineering Motivations

- Why do we need image interpolation?
 - We want BIG images
 - When we see a video clip on a PC, we like to see it in the full screen mode
 - We want GOOD images
 - If some block of an image gets damaged during the transmission, we want to repair it
 - We want COOL images
 - Manipulate images digitally can render fancy artistic effects as we often see in movies

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Scenario I: Resolution Enhancement



Low-Res.

High-Res.

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Scenario II: Image Inpainting



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Non-damaged Damaged

Scenario III: Image Warping



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Image Interpolation

- Introduction
 - What is image interpolation?
 - Why do we need it?
- Interpolation Techniques
 - 1D linear interpolation (elementary algebra)
 - 2D = 2 sequential 1D (divide-and-conquer)
 - Directional(adaptive) interpolation*
- Interpolation Applications
 - Digital zooming (resolution enhancement)
 - Image inpainting (error concealment)
 - Geometric transformations

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Upsampling

- This image is too small for this screen:
- How can we make it 10 times as big?
- Simplest approach:
 - repeat each row
 - and column 10 times
- (“Nearest neighbor interpolation”)

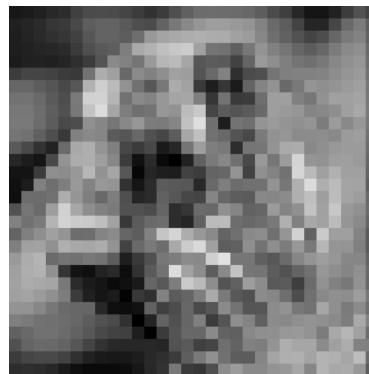
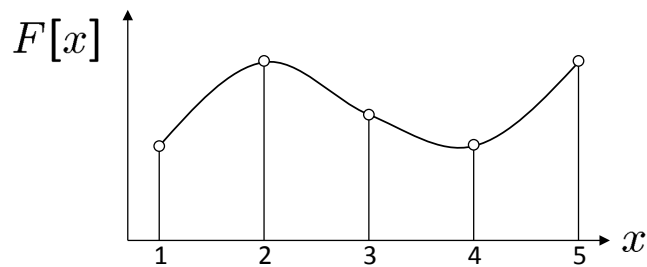


Image interpolation



$d = 1$ in this example

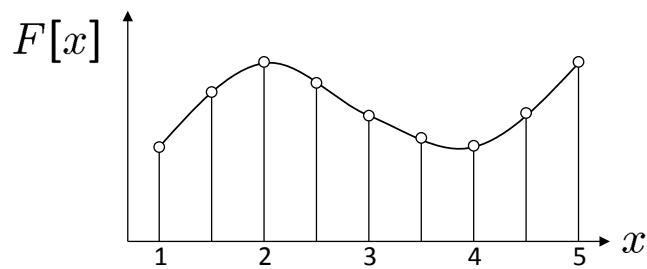
Recall how a digital image is formed

$$F[x, y] = \text{quantize}\{f(xd, yd)\}$$

- It is a discrete point-sampling of a continuous function
- If we could somehow reconstruct the original function, any new image could be generated, at any resolution and scale

Adapted from: S. Seitz

Image interpolation



$d = 1$ in this example

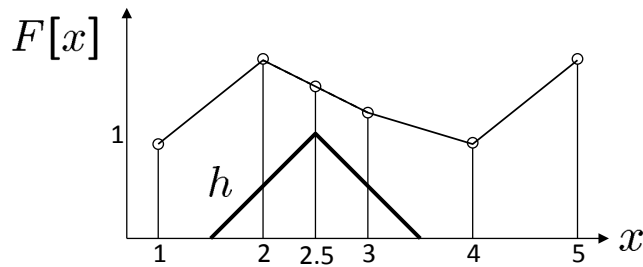
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Adapted from: S. Seitz

Image interpolation



$d = 1$ in this example

- What if we don't know f ?

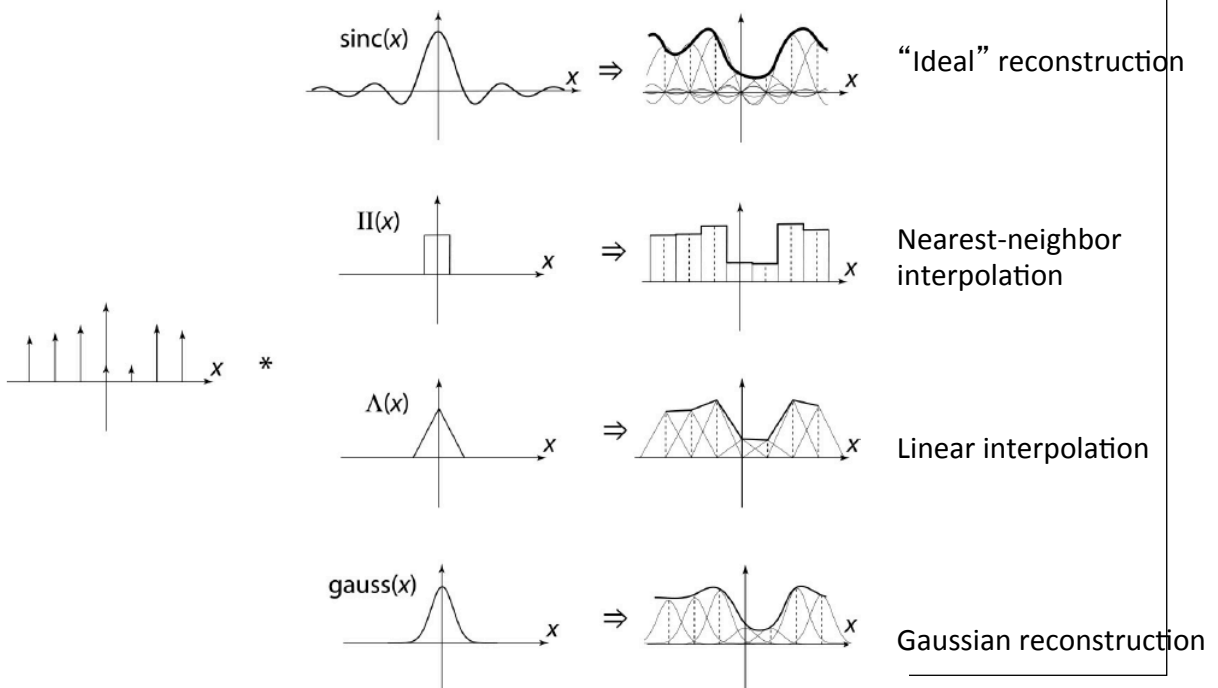
- Guess an approximation: \tilde{f}
- Can be done in a principled way: filtering
- Convert F to a continuous function:

$$f_F(x) = F\left(\frac{x}{d}\right) \text{ when } \frac{x}{d} \text{ is an integer, } 0 \text{ otherwise}$$
- Reconstruct by convolution with a *reconstruction filter*, h

$$\tilde{f} = h * f_F$$

Adapted from: S. Seitz

Image interpolation



Source: B. Curless

Ideal reconstruction

3.13 IMAGE INTERPOLATION ALGORITHMS

A digital image $f(n_1, n_2)$ may be thought of as a sampled region of an analog image $f(x, y)$ having continuous coordinates x, y :

$$f(n_1, n_2) = f(x, y) |_{x=n_1T_1, y=n_2T_2} \quad (3.13.1)$$

as has already been described in Chapter 1. T_1, T_2 are the sampling intervals along the x, y axes. If the analog image $f(x, y)$ is band-limited:

$$F(\Omega_1, \Omega_2) = 0 \text{ for } |\Omega_1| \geq \frac{\pi}{T_1}, |\Omega_2| \geq \frac{\pi}{T_2} \quad (3.13.2)$$

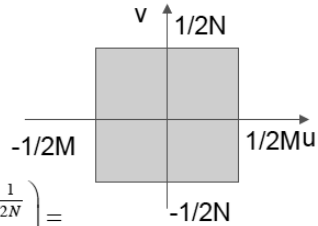
and the sampling frequencies are above the Nyquist frequencies, it can be recovered from the sampled image $f(n_1, n_2)$ by using the sinc interpolation formula [DUD84]:

$$f(x, y) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} f(n_1, n_2) \frac{\sin \frac{\pi}{T_1}(x - n_1T_1)}{\frac{\pi}{T_1}(x - n_1T_1)} \frac{\sin \frac{\pi}{T_2}(y - n_2T_2)}{\frac{\pi}{T_2}(y - n_2T_2)} \quad (3.13.3)$$

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Ideal reconstruction

$$\begin{aligned} h(x, y) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(u, v) e^{j2\pi(ux+vy)} du dv = \int_{\frac{1}{2N}}^{\frac{1}{2M}} \int_{\frac{1}{2N}}^{\frac{1}{2M}} MNe^{j2\pi(ux+vy)} du dv \\ &= \int_{\frac{-1}{2M}}^{\frac{1}{2M}} Me^{j2\pi ux} du \int_{\frac{-1}{2N}}^{\frac{1}{2N}} Ne^{j2\pi vy} dv \\ &= MN \frac{1}{j2\pi x} \left(e^{j2\pi x \frac{1}{2M}} - e^{-j2\pi x \frac{1}{2M}} \right) \times \frac{1}{j2\pi y} \left(e^{j2\pi y \frac{1}{2N}} - e^{-j2\pi y \frac{1}{2N}} \right) = \\ &= \frac{1}{\frac{\pi}{M} x} \frac{1}{2j} \left(e^{j\pi x \frac{1}{M}} - e^{-j\pi x \frac{1}{M}} \right) \times \frac{1}{\frac{\pi}{N} y} \frac{1}{2j} \left(e^{j\pi y \frac{1}{N}} - e^{-j\pi y \frac{1}{N}} \right) = \\ &= \frac{\sin\left(\frac{\pi}{M} x\right)}{\frac{\pi}{M} x} \times \frac{\sin\left(\frac{\pi}{N} y\right)}{\frac{\pi}{N} y} \end{aligned}$$



$\sin(x) = \frac{1}{2j}(e^{jx} - e^{-jx})$

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Ideal reconstruction

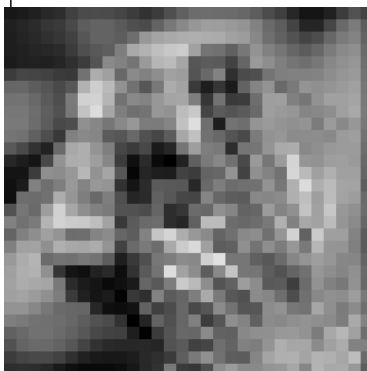
- The ideal reconstruction filter is a square window in the F-domain and a sinc function in the signal domain
- Its implementation is unpractical since it is prone to truncation artifacts and has relatively high computational complexity.
- Other low-pass filters can be used instead

Interpolation: LP filtering

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Image interpolation

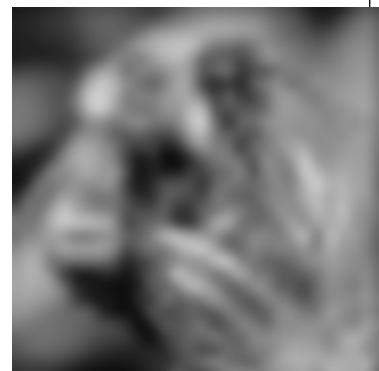
Original image:  x 10



Nearest-neighbor interpolation

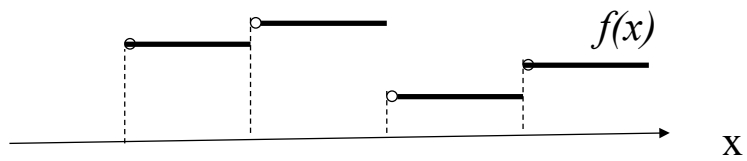
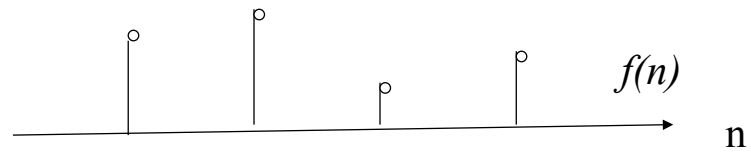


Bilinear interpolation



Bicubic interpolation

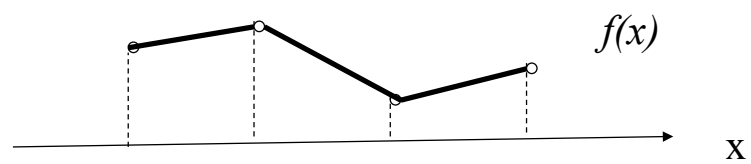
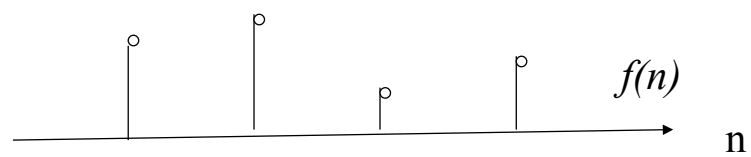
1D Zero-order (Replication)



$$h(x, y) = \begin{cases} 1 & -\frac{T_1}{2} \leq x \leq \frac{T_1}{2}, -\frac{T_2}{2} \leq y \leq \frac{T_2}{2} \\ 0 & \text{elsewhere in } [-T_1, T_1] \times [-T_2, T_2] \end{cases}$$

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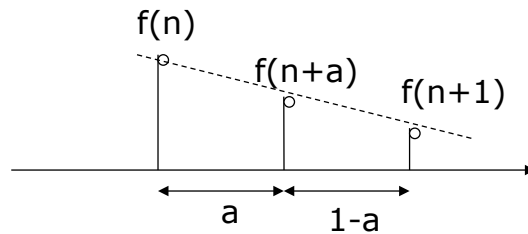
1D First-order Interpolation (Linear)



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Linear Interpolation Formula

Heuristic: the closer to a pixel, the higher weight is assigned
Principle: line fitting to polynomial fitting (analytical formula)



$$f(n+a) = (1-a)f(n) + af(n+1), \quad 0 < a < 1$$

Note: when $a=0.5$, we simply have the average of two

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Numerical Examples

$$f(n) = [0, 120, 180, 120, 0]$$

↓ Interpolate at 1/2-pixel

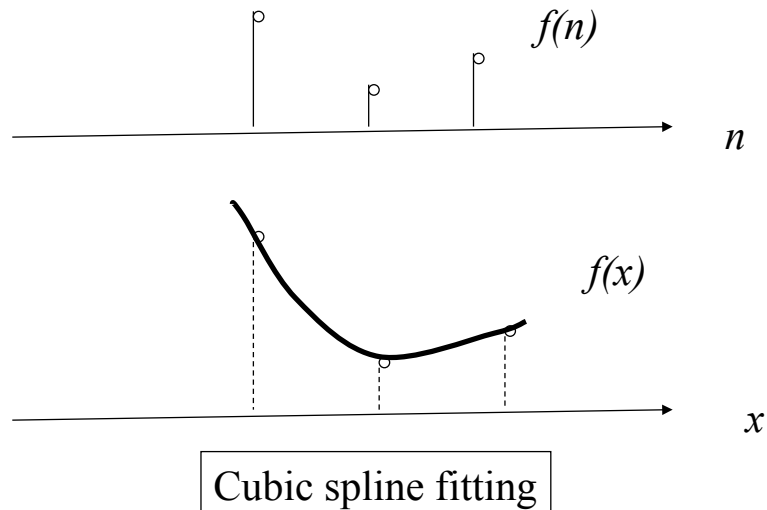
$$f(x) = [0, 60, 120, 150, 180, 150, 120, 60, 0], \quad x = n/2$$

↓ Interpolate at 1/3-pixel

$$f(x) = [0, 20, 40, 60, 80, 100, 120, 130, 140, 150, 160, 170, 180, \dots], \quad x = n/6$$

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1D Third-order Interpolation (Cubic)*



http://en.wikipedia.org/wiki/Spline_interpolation

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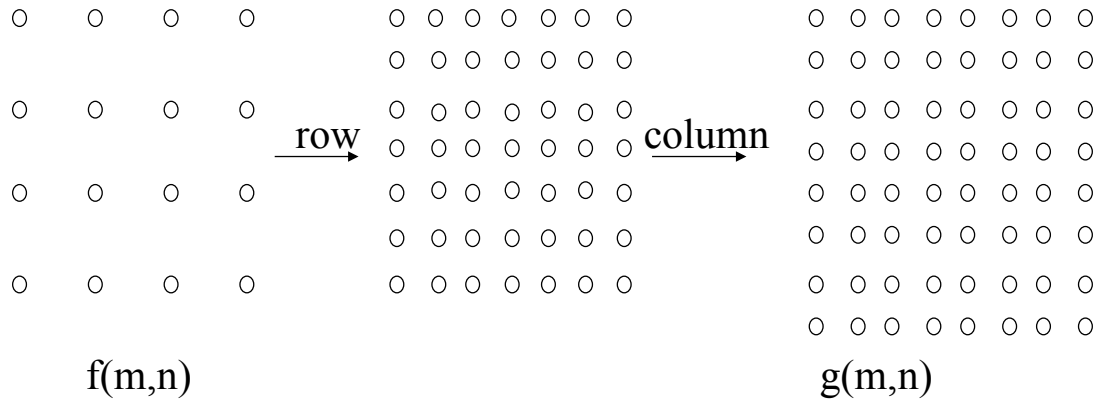
From 1D to 2D

- Engineers' wisdom: divide and conquer
 - 2D interpolation can be decomposed into two sequential 1D interpolations.
 - The ordering does not matter (row-column = column-row)
 - Such separable implementation is not optimal but enjoys low computational complexity

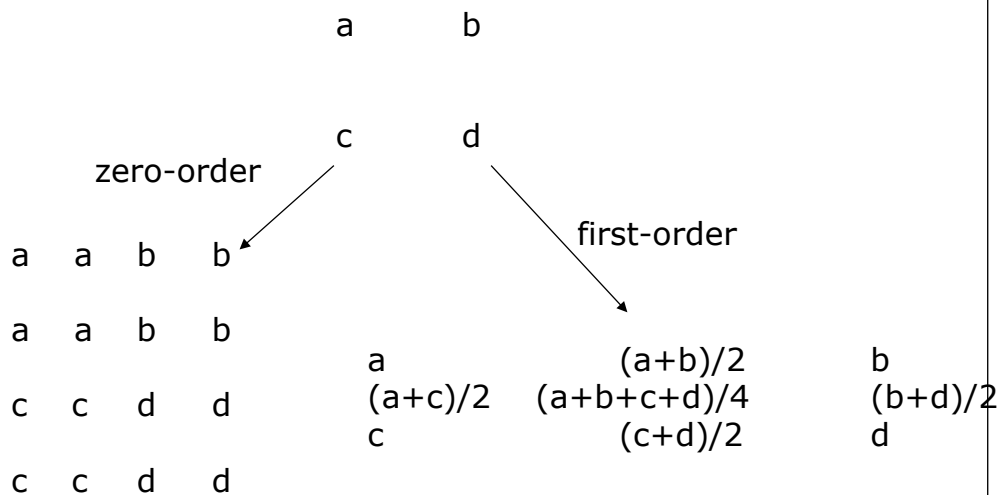
“If you don't know how to solve a problem, there must be a related but easier problem you know how to solve. See if you can reduce the problem to the easier one.” - rephrased from G. Polya's “How to Solve It”

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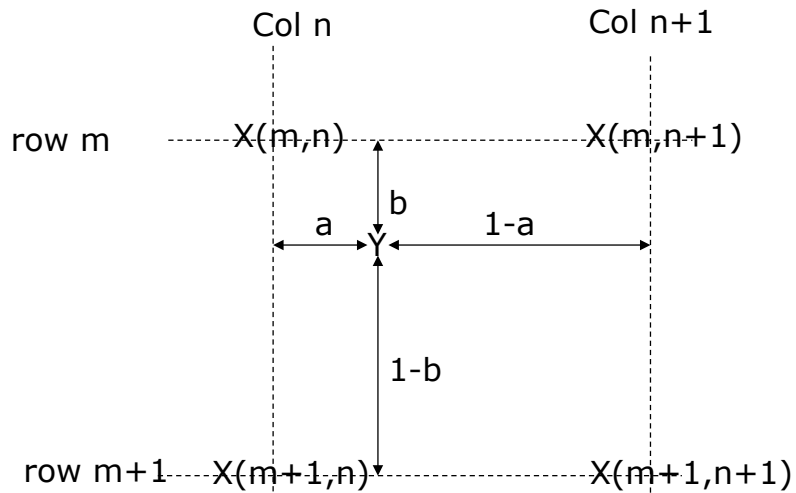
Graphical Interpretation of Interpolation at Half-pel



Numerical Examples



Numerical Examples (Con't)



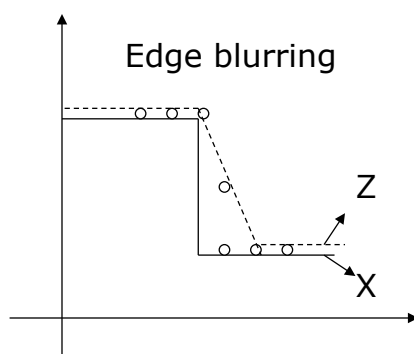
Q: what is the interpolated value at Y?

Ans.: $(1-a)(1-b)X(m,n) + (1-a)bX(m+1,n) + a(1-b)X(m,n+1) + abX(m+1,n+1)$

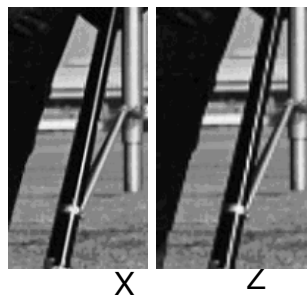
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Limitation with bilinear/bicubic

- Edge blurring
- Jagged artifacts



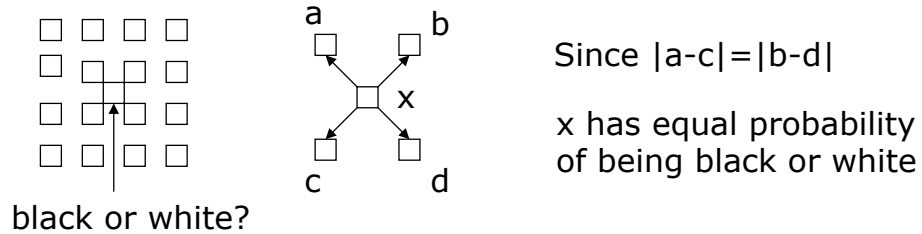
Jagged artifacts



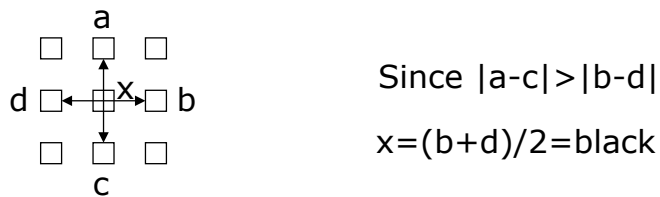
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Edge-Sensitive Interpolation

Step 1: interpolate the missing pixels along the diagonal



Step 2: interpolate the other half missing pixels



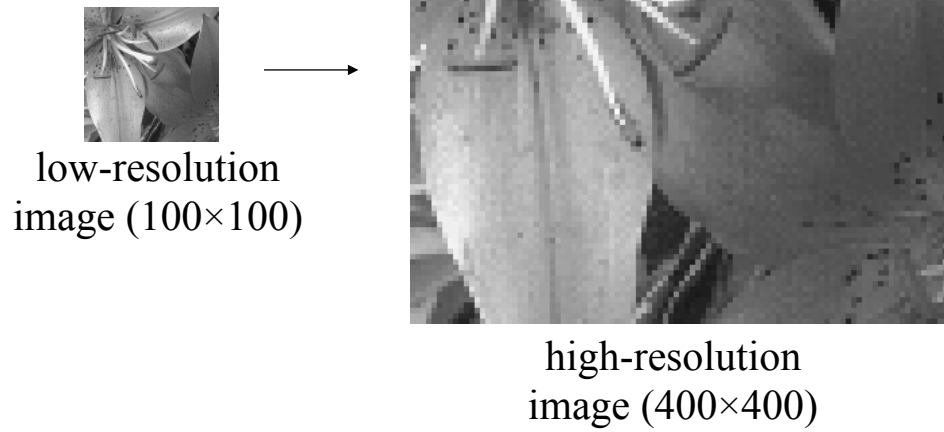
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Image Interpolation

- Introduction
- Interpolation Techniques
 - 1D zero-order, first-order, third-order
 - 2D zero-order, first-order, third-order
 - Directional interpolation*
- Interpolation Applications
 - Digital zooming (resolution enhancement)
 - Image inpainting (error concealment)
 - Geometric transformations (where your imagination can fly)

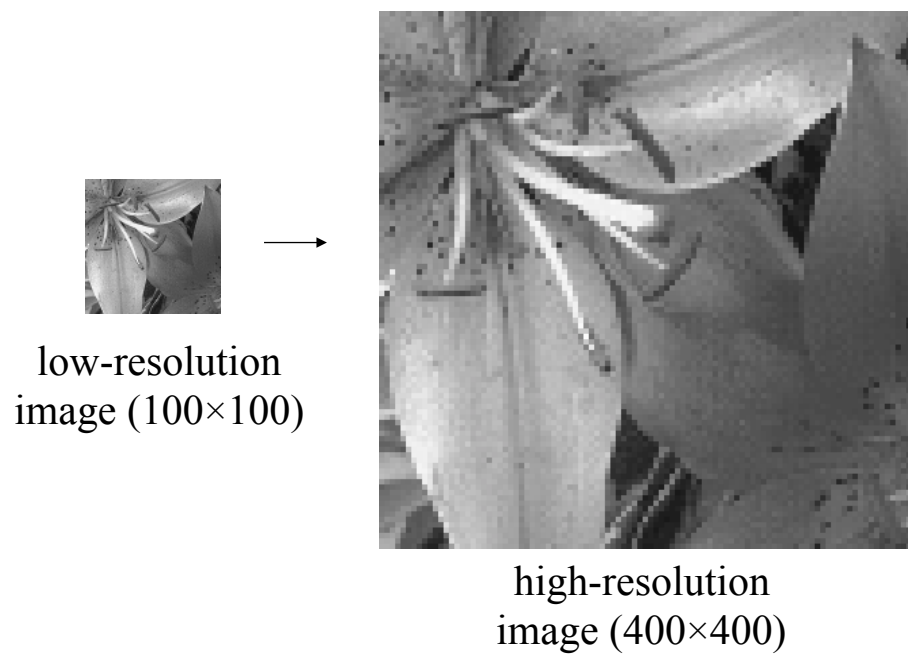
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Pixel Replication



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Bilinear Interpolation



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Bicubic Interpolation



low-resolution
image (100×100)



high-resolution
image (400×400)

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Edge-Directed Interpolation (Li&Orchard'2000)



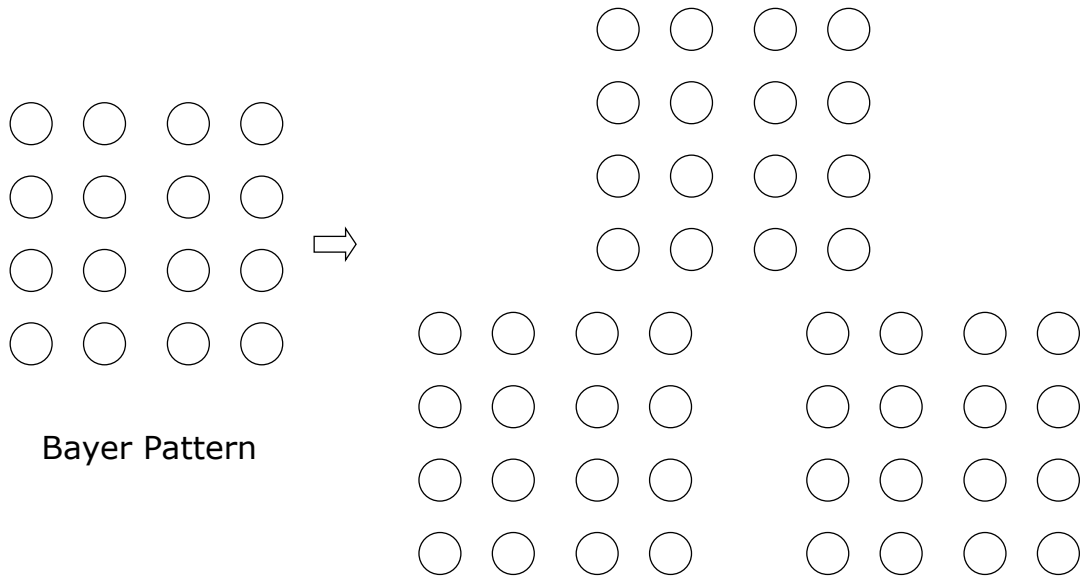
low-resolution
image (100×100)



high-resolution
image (400×400)

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Image Demosaicing (Color-Filter-Array Interpolation)



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Image Example



Ad-hoc CFA Interpolation



Advanced CFA Interpolation

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Error Concealment*



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Image Inpainting*



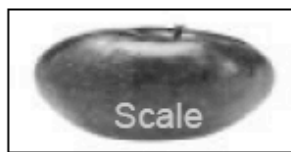
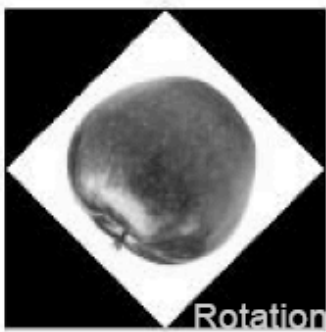
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Image Mosaicing*



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Geometric Transformation



MATLAB functions: griddata, interp2, maketform, imtransform

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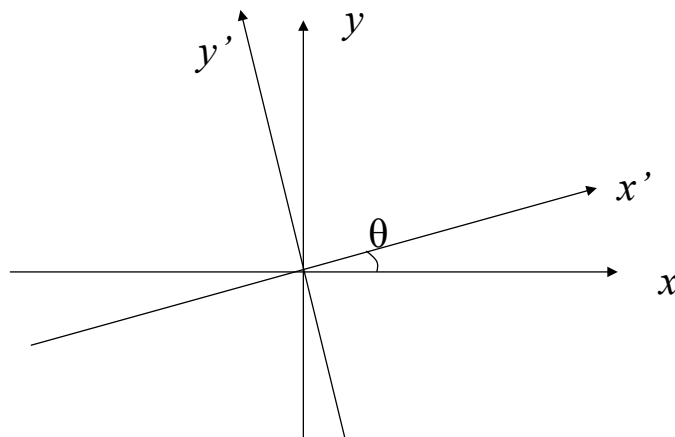
Basic Principle

- $(x,y) \rightarrow (x',y')$ is a geometric transformation
- We are given pixel values at (x,y) and want to interpolate the unknown values at (x',y')
- Usually (x',y') are not integers and therefore we can use linear interpolation to guess their values

```
MATLAB implementation: z' =interp2(x,y,z,x' ,y' ,method);
```

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Rotation



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

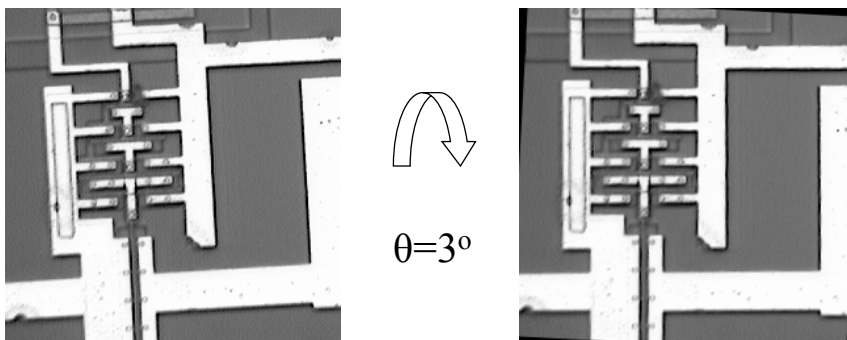
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MATLAB Example

```
z=imread('cameraman.tif');  
% original coordinates  
[x,y]=meshgrid(1:256,1:256);  
  
% new coordinates  
a=2;  
for i=1:256;for j=1:256;  
x1(i,j)=a*x(i,j);  
y1(i,j)=y(i,j)/a;  
end;end  
  
% Do the interpolation  
z1=interp2(x,y,z,x1,y1,'cubic');
```

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Rotation Example



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Scale



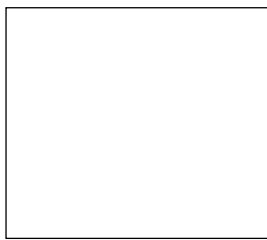
$a=1/2$ →



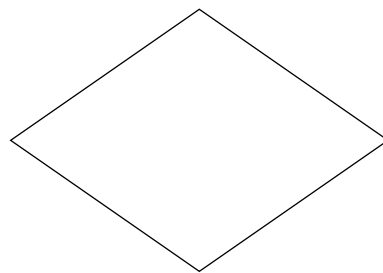
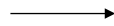
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & 1/a \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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Affine Transform



square



parallelogram

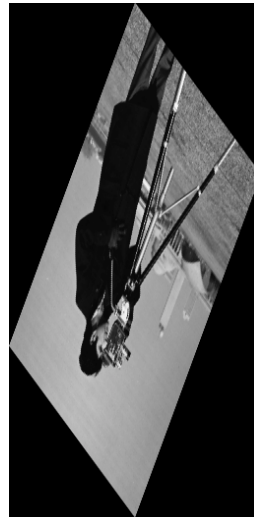
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

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Affine Transform Example

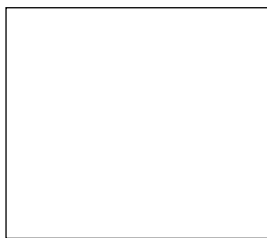


$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} .5 & 1 \\ .5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

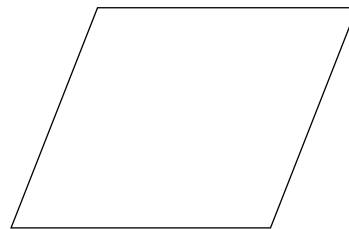
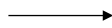


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Shear



square



parallelogram

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ s & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} d_x \\ d_y \end{bmatrix}$$

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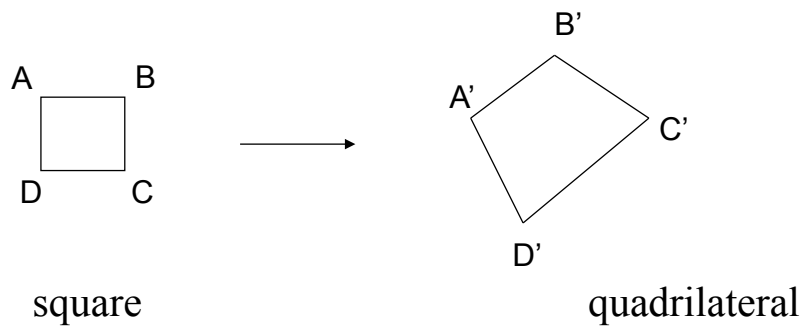
Shear Example



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ .5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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Projective Transform



square

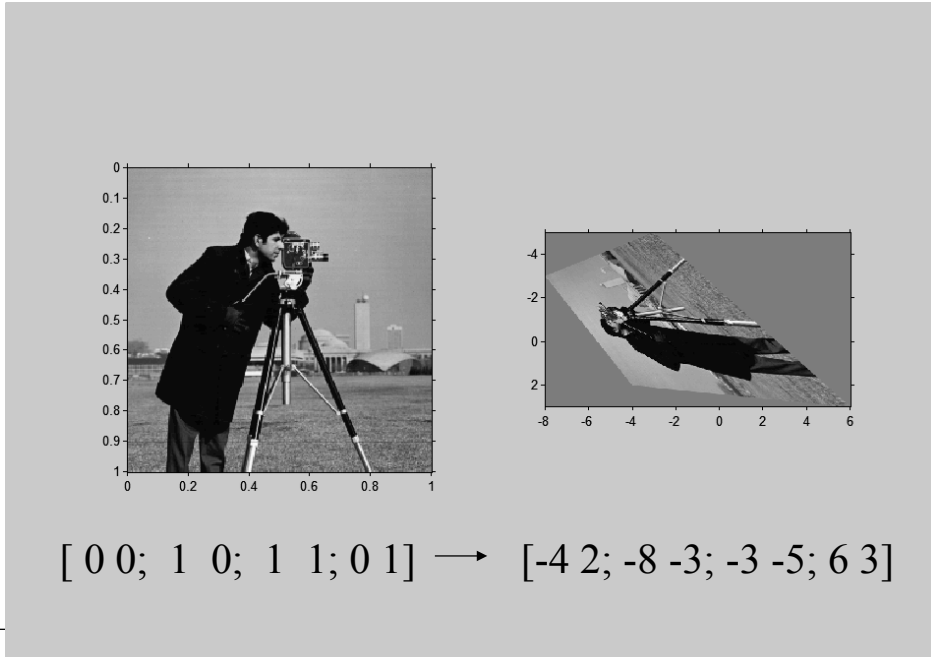
quadrilateral

$$x' = \frac{a_1x + a_2y + a_3}{a_7x + a_8y + 1}$$

$$y' = \frac{a_4x + a_5y + a_6}{a_7x + a_8y + 1}$$

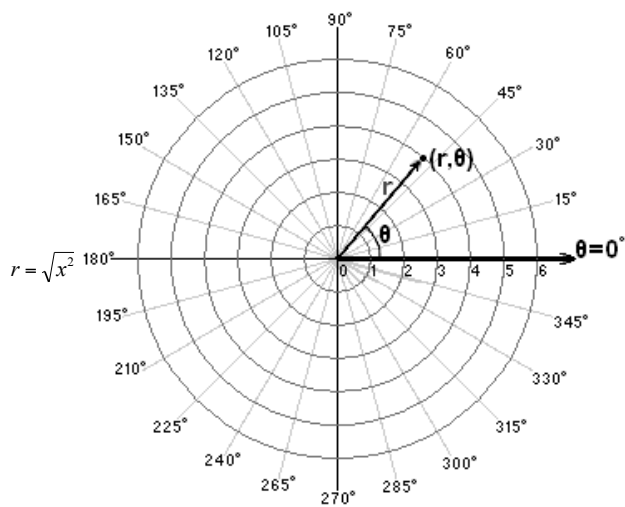
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Projective Transform Example

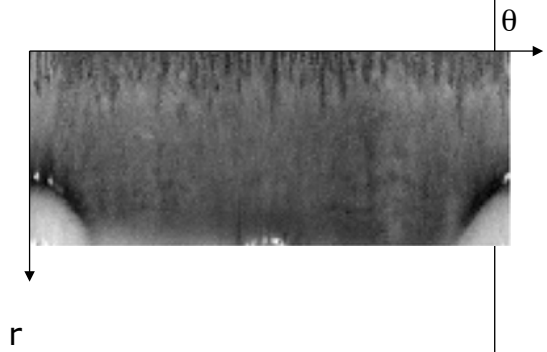
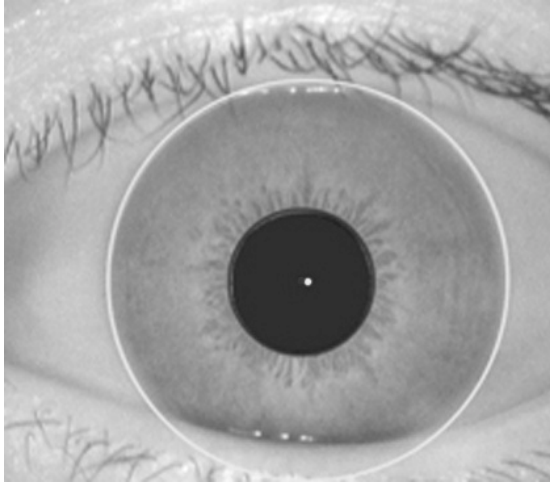


Polar Transform

$$\theta = \tan^{-1} \frac{y}{x}$$

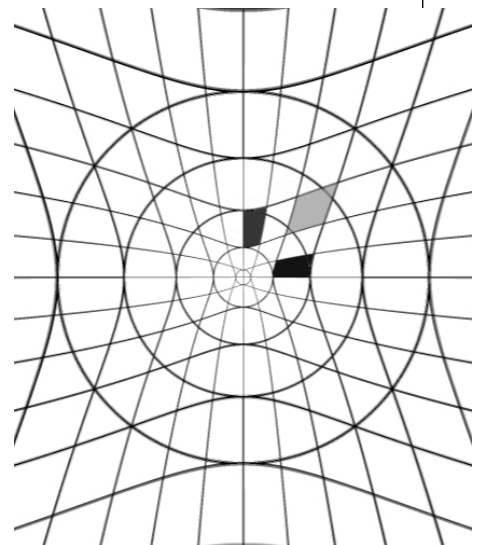
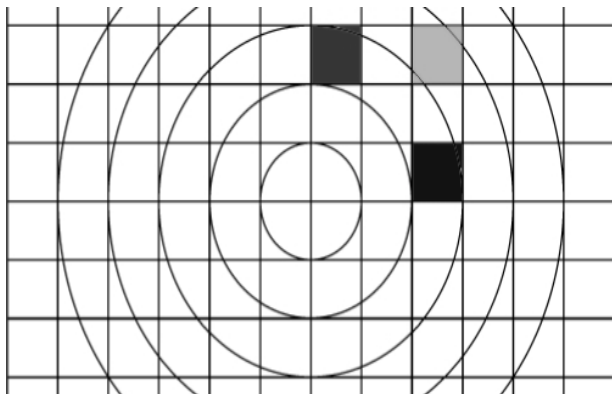


Iris Image Unwrapping

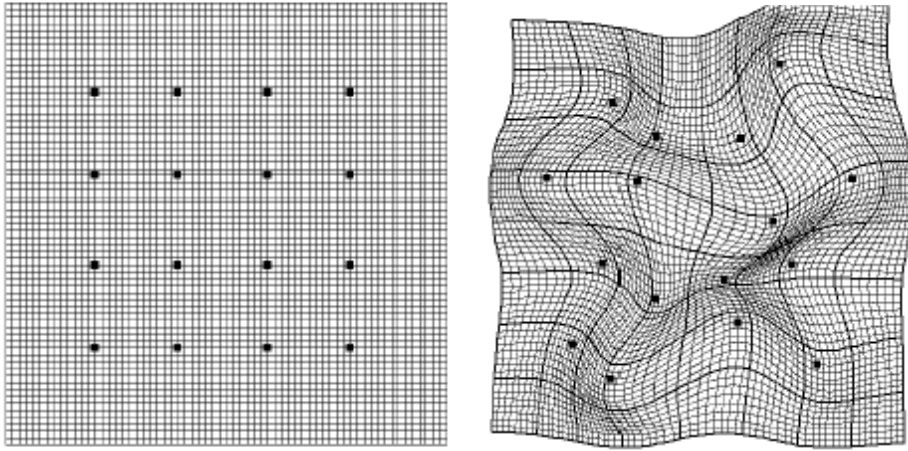


Use Your Imagination

$r \rightarrow \sqrt{r}$

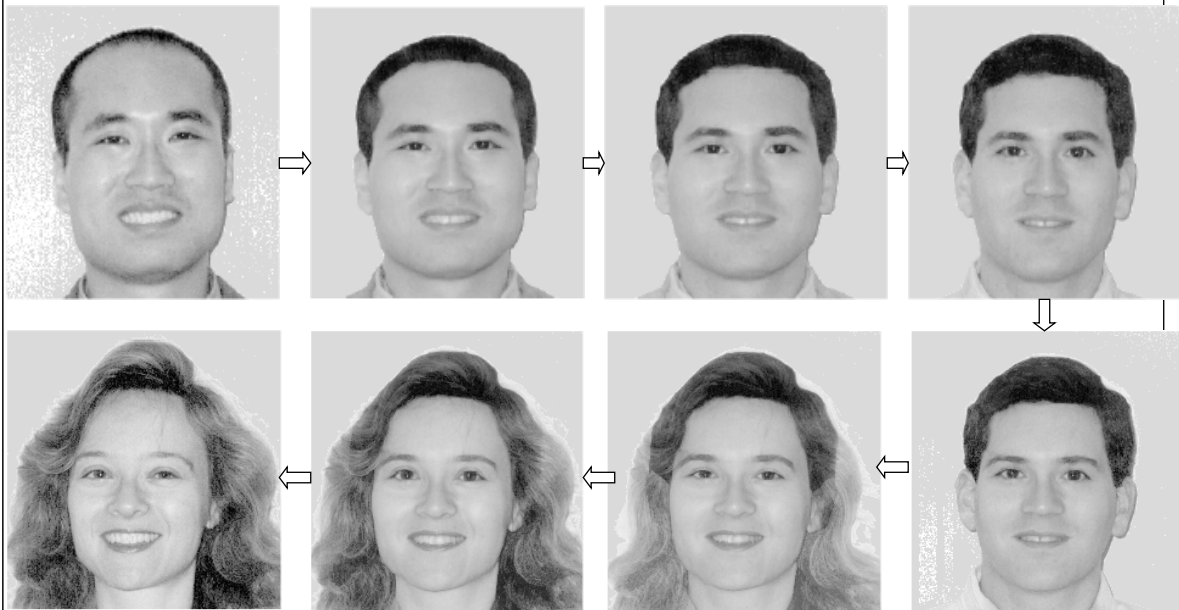


Free Form Deformation



Seung-Yong Lee et al., "Image Metamorphosis Using Snakes and Free-Form Deformations," *SIGGRAPH'1985*, Pages 439-448

Application into Image Metamorphosis



Summary of Image Interpolation

- A fundamental tool in digital processing of images: bridging the continuous world and the discrete world
- Wide applications from consumer electronics to biomedical imaging
- Remains a hot topic after the IT bubbles break