Considero

$$0 \leq \frac{(xy)^2}{x^2+y^2} \leq \frac{1}{2}\,\frac{x^4+y^4}{x^2+y^2} \overset{x^2y^2\geq 0}{\leq} \frac{1}{2}\,\frac{x^4+2x^2y^2+y^4}{x^2+y^2} = \frac{1}{2}\,\frac{(x^2+y^2)^2}{x^2+y^2} = \frac{x^2+y^2}{2}$$

$$0 \leq (x^2-y^2)^2 = x^4 - 2x^2y^2 + y^4$$

$(x,y)\to(0,0)$: 0 $\qquad$ $(x,y)\to(0,0)$: 0

$$\Rightarrow \lim_{(x,y)\to(0,0)} \frac{(xy)^2}{x^2+y^2} = 0$$

Quindi

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{1-\cos(xy)}{(xy)^2}\cdot\frac{(xy)^2}{x^2+y^2} = 0$$